THEORETICAL ANALYSIS OF ELECTRODES FOR MEASURING FIBRE ROTATION IN CARDIAC TISSUE

Peter R. Johnston and Barbara M. Johnston
School of Science
Griffith University
Kessels Rd
Nathan
Queensland
Australia 4111
E-mail: P.Johnston@griffith.edu.au

ABSTRACT
Four electrode techniques have long been used to determine conductivity parameters in cardiac tissue. This paper introduces a mathematical model and solution technique to theoretically analyse electrode configurations, specifically allowing for plunge electrodes. In particular, the focus is on using four electrode configurations to determine fibre rotation in cardiac tissue. Two configurations are analysed, the first with the four electrodes collinear and the second with one electrode removed from the line of the other three. It is found that the second electrode configuration can yield a value for the fibre rotation under the assumptions of the model.

KEY WORDS
Electromagnetic Field Simulation, Bidomain Model, Fibre Rotation, Electrodes.

1 Introduction
It has been shown previously that cardiac tissue anisotropy plays a vital role in several aspects of electrocardiography; for example, in modelling ST segment shift in subendocardial ischaemia [1, 2, 3] and in studies of defibrillation efficacy [4]. One part of this anisotropy comes from the differing electrical conductivity in the intra- and extracellular spaces, both longitudinally and transversely. Another part of this anisotropy comes from the fact that the sheets of fibres rotate relative to one another as they move from the epicardium to the endocardium.

Much work has been carried out to try to determine the above conductivity values [5, 6, 7]. Generally, these methods are based on the cable model of electric propagation through the tissue and the conductivity values are determined either from measurement of the amplitudes of the intracellular and extracellular action potentials, or by independently measuring the longitudinal and transverse action potential propagation velocities [8].

Another approach has been the use of a multi-electrode model to determine the conductivity values. The original idea of Plonsey and Barr [9] is to use a four electrode device placed on the epicardium, firstly along the fibres and secondly at right angles to them. The spacing of the electrodes is of the order of the length constant of the fibres. An extension of this idea by Le Guyader et al. [10] involves using a probe consisting of two orthogonal rows of four electrodes [11]. This probe was also placed on the epicardium and AC currents of varying frequency were injected. The conductivity parameters were calculated using a minimisation procedure which also yielded other electrical properties of the tissue, such as junction capacitance and junction conductivity. However, neither of these methods can determine the fibre rotation within the cardiac tissue.

Generally, fibre rotation has been determined from detailed morphological studies [12] or by actually slicing away the fibres layer by layer [13]. Such studies are difficult and time-consuming.

The recent advent of silver wire plunge electrodes [14], which can be placed in the cardiac tissue without causing significant injury currents, has provided the opportunity to employ new electrodes and new methodologies to determine the rotation of cardiac fibres. Here, two four electrode techniques are proposed and analysed theoretically as potential methods for finding the degree of fibre rotation. The first consists of one needle, containing four electrodes, which is injected into the cardiac tissue. The second is based on placing three needle electrodes into the tissue, two electrodes providing fixed current sources and the third measuring the resulting potential at the same level as the current sources as well as at the epicardium.

In the next section, a mathematical model, which takes account of many features of the cardiac tissue, is proposed and a solution method is discussed. The following section presents results of the numerical simulation as well as a methodology for using these electrodes to determine the fibre rotation. There is also a discussion of the potential of these methods. Finally, there is a discussion of the limitations of the mathematical model and some future directions.
2 Methods

2.1 Governing Equations

Since the idea of the proposed method is to measure the local fibre rotation, it is assumed that the cardiac tissue can be represented by a block of tissue, finite in the $x$ and $y$ directions with a length of $2L$ in each direction. It is also assumed that the epicardium is represented by the plane at $z = 0$ and the endocardium is represented by the plane at $z = 1$, which is in contact with a volume of blood extending to infinity in the positive $z$ direction.

A bidomain model [15, 16, 17] is used to account for the intracellular and extracellular regions of the tissue. The bidomain equations for the intracellular and extracellular potentials, $\phi_i$ and $\phi_e$, respectively are

$$\nabla \cdot (M_i \nabla \phi_i) = \beta / R_m (\phi_i - \phi_e)$$

(1)

$$\nabla \cdot (M_e \nabla \phi_e) = -\beta / R_m (\phi_i - \phi_e) - I_s$$

(2)

where $\beta$ is the surface to volume ratio of the cells, $R_m$ is the specific membrane resistance, $I_s$ is an external current source per unit volume and $M_i$, $M_e$ are conductivity tensors which reflect anisotropy in the cardiac tissue.

Finally, in the blood, which is a source-free region, the electric potential, $\phi_b$, is governed by Laplace’s equation

$$\nabla^2 \phi_b = 0$$

(3)

2.2 Conductivity Tensor

The electrical anisotropy of cardiac tissue comes from the fibrous nature of the tissue, with electrical conductivity being greater along the fibres than across them. Hence, four conductivity values are required: $\sigma_i^l, \sigma_i^t, \sigma_e^l, \sigma_e^t$, where the superscripts $i$ and $e$ refer to the intra- and extracellular domains respectively, and the subscripts $l$ and $t$ refer to the longitudinal and transverse directions respectively.

It will be assumed that the rotation of the fibres varies linearly with depth [18] and that the fibre layers are parallel to the epicardium [13]. These assumptions imply that the conductivity tensors can be written as

$$M_k(x, y, z) = \begin{pmatrix}
(\sigma_i^k - \sigma_e^k) c^2 + \sigma_i^k & (\sigma_i^k - \sigma_e^k) c s & 0 \\
(\sigma_i^k - \sigma_e^k) c s & (\sigma_i^k - \sigma_e^k) s^2 + \sigma_i^k & 0 \\
0 & 0 & \sigma_e^k
\end{pmatrix}$$

(4)

where $k = i$ or $e$, $c = \cos \psi(z)$ and $s = \sin \psi(z)$ and $\psi(z) = \alpha z$, where $\alpha$ is the total fibre rotation angle through the tissue which lies between 0 and $180^\circ$ [12]. The model assumes that the fibres on the epicardium are aligned with the positive $x$-axis and the fibre sheets rotate anticlockwise from the epicardium to the endocardium.

2.3 Boundary Conditions

To solve the differential equations (1), (2) and (3) a set of boundary conditions is required. Assuming that the epicardium is insulated means that

$$\frac{\partial \phi_e}{\partial z} = \frac{\partial \phi_i}{\partial z} = 0$$

(5)

Further, at the interface between the tissue and the blood, there is continuity of extracellular potential and current, but the intracellular space is insulated by the extracellular space [19]; that is,

$$\phi_e = \phi_b, \sigma_b \frac{\partial \phi_b}{\partial z} = \sigma_i \frac{\partial \phi_i}{\partial z} = 0$$

(6)

Since the blood mass is assumed infinite in the positive $z$-direction, $\phi_b \to 0$ as $z \to \infty$. Finally, the $x$ and $y$ boundaries of the domain are insulated so the derivatives of $\phi_e$ and $\phi_i$ in the $x$ and $y$ directions at these boundaries are zero.

2.4 Solution Method

Assume $\phi_e$ and $\phi_i$ are given by

$$\phi_k(x, y, z) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} C^k_{nm}(z) \cos m \pi y \cos n \pi x$$

$$+ D^k_{nm}(z) \sin m \pi y \cos n \pi x$$

$$+ E^k_{nm}(z) \cos m \pi y \sin n \pi x$$

$$+ F^k_{nm}(z) \sin m \pi y \sin n \pi x$$

(7)

for $k = i$ or $e$ and substitute these into the differential equations. This gives a coupled system of ordinary differential equations in $z$ for the coefficients $C^k_{nm}(z)$, $D^k_{nm}(z)$, $E^k_{nm}(z)$ and $F^k_{nm}(z)$. These ordinary differential equations need to be solved numerically and this is achieved using a simple one dimensional finite difference method. Application of the finite difference method yields a banded system of linear algebraic equations which are solved using standard techniques [20] to give the required coefficients. Once the coefficients are found, the potentials can be obtained by summation of the above series.

The advantage of this approach over a full numerical approach or a Fourier Transform approach [1, 2, 10] is that the potentials are calculated only at points at which they are required. In particular, it is only necessary to calculate the potentials at the measuring electrodes.

2.5 Electrode Configurations

The idea of the paper is to theoretically analyse electrode placements to determine fibre rotation in cardiac tissue. As a simple first example, consider a single needle electrode, as shown in Figure 1. This electrode is inserted into the cardiac tissue normal to the epicardium, and consists of two current source electrodes and two electrodes
measuring the potential difference. This electrode is the direct analogue of the electrode proposed by Plonsey and Barr [9], except that here it is inserted into the cardiac tissue instead of it lying along the surface. For the purpose of the simulations presented here, the distance between the current injection electrodes was set at 4 mm with the potential measurement electrodes 3 mm apart, placed evenly between the current electrodes. The top current injection electrode was placed 3 mm below the epicardium.

As a second example, consider a new four electrode configuration. The configuration consists, firstly, of two current sources placed on needle electrodes and inserted a fixed distance into the cardiac tissue (Figure 2). Secondly, two further electrodes, inserted into the tissue on another needle, are used to record the electric potential generated by the current source electrodes. The lower of these two electrodes is placed at the same level as the current injection electrodes and the upper electrode is placed at the level of the epicardium. This electrode configuration differs from previous four electrode configurations [9, 10] in that the four electrodes are no longer collinear. Here, only three of the electrodes are collinear with the fourth fixed to the epicardium. For the simulations performed, the two current electrodes were placed a distance of 4 mm apart. The measurement needle was between the first two needles, 0.5 mm from the left.

The two models proposed above will be used to study the effects of fibre rotation on the potential difference between the two measuring electrodes in each configuration.

### 2.6 Modelling Parameters

The method outlined here is based on the assumption that the longitudinal and transverse electrical conductivity values in both the intra- and extracellular spaces is known already. For the purpose of the simulations presented here, these conductivity values are taken from Clerc [5]; that is, \( \sigma_i^l = 0.00174 \, \text{S/cm} \), \( \sigma_i^c = 0.00625 \, \text{S/cm} \), \( \sigma_e^l = 0.000193 \, \text{S/cm} \) and \( \sigma_e^c = 0.00236 \, \text{S/cm} \). The conductivity of blood, \( \sigma_b \), is taken to be 0.0067 S/cm. The block of tissue modelled was 1 cm thick and 2 cm in each of the \( x \) and \( y \) directions (that is, \( L=1 \)). The values of \( R_m=9100 \, \text{\Omega cm}^2 \) and \( \beta=2000 \, \text{cm}^{-1} \) are those used by Le Guyader et al. [10], since these values are not available for Clerc’s work. A 2 \( \mu \text{A} \) current \( I_s \) is applied.

### 3 Results and Discussion

Figure 3 shows the potential measured along the single needle electrode described above. Although the potentials shown are continuous, the electrode would only measure the potentials at the extremes of the data plotted. This figure shows that for each fibre rotation considered, there is no difference in the potentials recorded along the electrode. Hence, this type of electrode would be of no use in determining the fibre rotation through the cardiac tissue. This is perhaps not surprising as any needle inserted in the fashion described above would form an axis of rotation for the fibres.

Figure 4 shows plots of the potential difference on the measurement electrodes for the three needle electrode model, plotted as a function of fibre rotation. The maximum fibre rotation considered was 180°, as this was believed to be the highest degree of fibre rotation observed [12]. Each line in the plot corresponds to a different depth of the current electrodes. The plot shows two distinct types of lines. When the electrodes are inserted to a point in the upper half of the tissue (\( z < 0.5 \)), then the curve is a monotonically increasing function of the degree of fibre rotation. However, when the electrodes are inserted to a point in the lower half of the tissue (\( z > 0.5 \)), then the curve reaches a maximum and starts to become more negative again. From this figure it can be observed that if the electrodes were inserted to a point in the upper half plane and a potential difference recorded, then a unique value for the fibre rota-
Figure 3. Potentials along a single needle electrode for varying degrees of fibre rotation.

Figure 4. Potential differences measured using the three needle electrode approach for varying degrees of fibre rotation and depths of the current injection electrodes.

4 Conclusions

This paper has presented an in-principle method for determining cardiac tissue fibre rotation, based on a mathematical model of the cardiac tissue and a four electrode measuring system using three needle electrodes. The idea is to use the model to determine a plot of potential difference versus fibre rotation at a given depth of the source electrodes, then measure the potential difference using the electrode configuration described and read off the fibre rotation from the plot.

The proposed method does have some limitations; the principal one is that the method can only be applied in the region of the ventricular wall where the fibre sheets are almost parallel to the epicardium [13]. However, near the base and the apex of the heart, this assumption is no longer valid. Secondly, it may not be the case that the fibre rotation varies linearly with depth; however, this is a reasonably common assumption used in modelling [18]. To overcome this problem, it may be necessary to make recordings at more than one depth.

It might well appear that the three needle approach presented here would be difficult to implement in practice. However, as mentioned above, the single electrode would not supply the required information. A compromise might be to use a single electrode injected at an oblique angle to the epicardium.

Acknowledgement: This work was funded by the Australian Research Council.

References


