DETERMINATION OF EEG SLEEP SPINDLE FREQUENCY
WITH DFT AND MATCHING PURSUIT APPROACHES

Eero J. Huupponen, Antti J. Saastamoinen, Jussi T. Virkkala, Sari-Leena J. Himanen, Joel Hasan, and Alpo O. Värri

1Institute of Signal Processing, Tampere University of Technology, Finland
{eero.huupponen, antti.saastamoinen, alpo.varri}@tut.fi
2Department of Clinical Neurophysiology, Tampere University Hospital, Finland
{sari-leena.himanen, jussi.virkkala}@pshp.fi
3Section of Clinical Neuroscience, Finnish Institute of Occupational Health, Helsinki, Finland
joel.hasan@pshp.fi, joel.hasan@ttl.fi

ABSTRACT
In the present work, sleep EEG of four healthy subjects was analyzed, containing a total of 3974 visually scored spindles. Frequency of each spindle was determined with DFT and Matching Pursuit (MP) approaches. The estimated spindle frequencies were similar by the two approaches. Measures to quantify the dominance of spindle activity were developed. Results seem to indicate that there is one single dominant spindle activity component in about 90% of the spindles. In all these cases, it seems to be justified to use FFT and zero padding to resolve the spindle frequency more accurately than with no zero padding. In MP approach the frequency resolution can be selected freely.

KEY WORDS
Sleep EEG, sleep spindles, DFT, Matching Pursuit

1. Introduction
EEG oscillations during sleep are generated in thalamo-cortical networks that are mutually connected [1]. Sleep spindles are sleep-maintaining events that are seen in large numbers in sleep EEG as short-lasting waveforms (please see Figure 1.a), typically with 0.5-1 s duration [2,3]. Frequencies of individual spindles (10.5-16 Hz) typically decrease along with deepening sleep and increase as the sleep gets lighter [4]. Sleep spindles occurring in frontal brain regions have been reported to be generally of lower frequency than those in central brain regions [4,5,6].

Computer-based methods aim to provide an objective and accurate description of sleep process and sleep micro-events [7,8]. Therefore, as precise as possible determination of spindle frequency is important. Clearly, a good time resolution is needed to analyze such a short event. This is challenging as the frequency resolution should be as high as possible, too. The fundamental frequency resolution of Discrete Fourier Transform (DFT) of a 0.5 s long signal segment is very modest. It has been stated that “the quantization error” in the accuracy of estimating the frequency of a spectral peak can be reduced by employing zero padding in FFT calculus while the fundamental frequency resolution is not improved [9]. If there are multiple spectral peaks close to each other, these may merge providing non-correct frequency estimates. The aim of the present work was to study how accurately real-life spindle frequency could be determined with classical frequency domain approach of DFT and the relatively new time-domain based Matching Pursuit (MP) approach [6,10].

2. Methods
Sleep EEG from two females and two males of median age 35 years (range 25-43 years) were studied. Total duration of the recordings was 33 h 2 min. There were a total of 3974 spindles in these data, visually scored by a medical doctor on the C4-A2 channel, which was also automatically analyzed in the present work. The EEG sampling rate \( f_s \) was 200 Hz.

Discrete Fourier Transform (DFT) describes the time domain signal as sum of sinusoidal waves and is therefore a heuristically good method for sleep EEG analysis (containing plenty of sinusoidal type signal components). In DFT, the frequency resolution is \( \Delta f = f_s / M \), where \( f_s \) is the sampling frequency and \( M \) the total number of samples. The DFT amplitude spectrum calculated without zero padding is called the fundamental DFT in the following text.

Analysis A), spindle peak sharpness

Raw EEG signal was analyzed in analysis A). The applied windowing was such that at each spindle second (denoted as \( j \)), five partly overlapping 0.5 s (\( M=100, \beta=3 \)) Saramäki window functions [11] were used, centred at 0.1, 0.3, 0.5, 0.7 and 0.9 s. The five amplitude spectra were then extracted and the one showing the largest peak amplitude in spindle band of 10.5-16 Hz was used in subsequent analysis. The frequency resolution of the fundamental DFT, denoted as \( \Delta f_{DFT} \), was 2 Hz. By studying the two bins sur-
rounding the peak amplitude (Figure 1.b), a measure of spindle peak sharpness, denoted as $P_s$, was developed,

$$P_s = \frac{A_p}{(0.5\times(A_{p-1} + A_{p+1}))},$$

where $A_p$ is the peak amplitude, and $A_{p-1}$ and $A_{p+1}$ are the two surrounding amplitudes. The narrower and higher the peak is, the larger values $P_s$ gets. If the spindle peak $A_p$ is accompanied by other large peaks in the spindle band (other spindle activity components), the values of $P_s$ are near to one. The values of $P_s$ determined at each of the 3974 spindles are denoted as $P_s[i], j=1,..., 3974$.

**Analysis B), spindle frequency comparison**

Matching Pursuit (MP) and FFT approaches were both used to determine the frequency of each spindle in analysis B). Band-pass filtered EEG was used to allow a fair comparison between FFT and MP approaches, with pass band of 10.5-16 Hz (-3 dB at 10.0 and 16.5 Hz) and at least 40 dB attenuation in stop bands. This filtering was first used only in the present MP routine, but since sleep EEG occasionally contained plenty near 10 Hz activity, it was better to use exactly the same signal in this comparison.

Same windowing was used as in analysis A) with five partly overlapping 0.5 s long windows. In FFT based spindle frequency determination, Saramäki window functions and then zero padding to 512 samples was used this time (frequency resolution, denoted as $\Delta f_{DFT}$, was 0.39 Hz). The amplitude spectrum with largest peak amplitude provided the spindle frequency, denoted as $f_{DFT}[j], j=1,..., 3974$.

Also a Matching Pursuit (MP) based routine was applied. As there was only the spindle frequency band (10.5-16 Hz) left in the filtered EEG signal, the frequency of the first and thus largest MP component provided the spindle frequency estimate. The routine made a time domain fitting, using sinusoidal signals modulated by three different modulating functions. Gaussian distribution function $f(x)$ was used

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$  

In the middle of a spindle, the amplitude envelope may remain rather constant or form a classical belly shape. Therefore, three modulating functions $g[k], i=1,2,3,$ were used. The $g_1$ was chosen as constant amplitude, $g_1[k]=1, k=1,2,...,100$, $g_2$ was determined so that it formed a belly to reduce by 50% from middle to the ends by selecting $g_2[k]=f(k*0.02), \mu=1$ and $\sigma=0.75, k=1,2,...,100$, $g_3$ was determined so that it reduced by 100% from middle to the ends by selecting $g_3[k]=f(k*0.02), \mu=1$ and $\sigma=0.3, k=1,2,...,100$. The frequency of the sinusoid $s(t)=\cos(2\pi t/\varphi)$, denoted as $f$, ranged from 10.5 to 16.0 Hz at 0.1 Hz intervals. The phase shift $\varphi$ ranged from 0 to $2\pi$ with increments of 0.05 rad. Thus, there were a total of 1120 slightly different sinusoids $s[k], k=1,2,...,100$. All sinusoids were modulated by $g[k], i=1,2,3$, and scaled as follows: $c[k]=g[k]^s[k], \Sigma|c[k]|/100=1, k=1,2,...,100$, totaling 3360 code vectors of 0.5 s duration (100 samples).

An example of all three types of code vectors are shown in Figure 2. The spindle frequency was then determined by comparing the filtered EEG signal to code vectors $c$. The 0.5 s long EEG segment $eeg[k]$ was scaled so that $\Sigma|eeg[k]|/100=1, k=1,2,...,100$ (no Saramäki windowing). This segment was compared to the code vectors by computing the Euclidian distance measure $d$ for each code vector $c$ as

$$d = \sum_{i=1}^{100} (c[i]-eeg[i])^2.$$  

At each spindle second this was repeated for each five partly overlapping signal segments. The overall minimum Euclidian distance, denoted as $d_{\text{min}}$ provided the code vector $c_{\text{min}}$ with best match to current spindle and spindle frequency, denoted as $f_{\text{min}}[j], j=1,..., 3974$, was obtained as the frequency of that code vector.

**Figure 2. Example of the three types of code vectors with different amplitude envelopes. The duration of each was 0.5 s (100 samples). The sinusoid $s[i]/(f=13.0 \text{ Hz}, \varphi=0 \text{ rad})$ was modulated by $g_1[i]$ (a), $g_2[i]$ (b) and $g_3[i]$ (c), resulting in a constant sinusoidal amplitude, one damping by 50% and one by 100% from the middle to the ends, respectively.**
denoted as $G_{c}$, was quantified as the value $d_{\text{min}}$ compared to a squared sum of the respective code vector $c_{\text{min}}$

$$G_{c}[j] = 1 - \frac{d_{\text{min}}}{\sum_{i=1}^{100}(c_{\text{min}}[i])^2}.$$ 

Large values of $G_{c}[j]$ indicated a good fitting so that value one would be a perfect match. Respectively, small values indicated a poor MP fitting.

The estimated spindle frequencies with MP and FFT approaches at each spindle were compared by extracting their differences, denoted as $\Delta[f_j] = f_{\text{MP}}[j] - f_{\text{FFT}}[j], j=1,\ldots, 3974$.

Analysis C), simulated test signal

A simulated “spindle” of a pure 13.5 Hz sinusoid was used to illustrate the idea of “the quantization error” of frequency estimation by comparing the fundamental DFT and 512-point zero padded FFT.

3. Results

Analysis A). The values of $P_s[f_j]$ were larger or equal to 1.25 in 90% of all spindles. These large values of $P_s[f_j]$ seemed to indicate a narrow peak (it depended also on the frequency of the spindle peak with respect to locations of bins at 2 Hz steps). The goodness of MP fitting resulted in a similar outcome so that values of $G_{c}[j]$ were larger than 0.80 in 91% of all spindles.

Analysis B). Frequencies of individual spindles by FFT and MP methods were overall close to each other. The probability density functions of the determined spindle frequencies are shown in (Fig. 3a). The PDFs were similar above 13 Hz while slightly different below 13 Hz.

In the comparison of the frequency values by their differences $\Delta[f_j] = f_{\text{MP}}[j] - f_{\text{FFT}}[j]$, the PDF of $\Delta[f_j]$ formed a narrow peak centered at 0 Hz (Fig. 3b). Median frequency difference was 0.0 Hz and the mean difference was -0.07 Hz indicating slightly higher mean spindle frequency by FFT method than MP.

Figure 3. PDFs of estimated spindle frequencies $f_{\text{MP}}[j]$ and $f_{\text{FFT}}[j]$ (a) and corresponding frequency differences $\Delta[f_j]$ at all spindles, $j=1,\ldots, 3974$ (b).

Analysis C). The fundamental DFT and the 512-point zero padded FFT of the simulated pure 13.5 Hz sinusoid are seen in Figure 4. The zero padded FFT revealed the 13.5 Hz spectral peak more accurately than the fundamental DFT, as the value of $f_{\text{FFT}}[j]$ was 14.0 Hz and $f_{\text{FFT}}[j]$ was 13.67 Hz. The additional points in 512-point FFT were not mere interpolations from one bin to the next of fundamental DFT. Thus, in this simulated dominant “spindle”, zero padding to 512-points reduced the quantization error of frequency estimation from 0.5 Hz to 0.17 Hz.

Let us further consider the “quantization error” reduction in frequency estimation achieved via zero padding. Let $f_j$ denote the true frequency of spindle and $f_j$ the estimated spindle frequency in this simulated case, where there is only one major “spindle” peak. The quantization error $e_j$ is then $e_j = f_j[f] - f_j$. The maximum quantization error is $\Delta[f] / 2$ and thus

$$|e_j| \leq 1 \text{ Hz using the fundamental DFT},$$

$$|e_j| \leq 0.2 \text{ Hz with FFT and zero padding to 512 samples},$$

$$|e_j| \leq 0.05 \text{ Hz with FFT and zero padding to 2048 samples.}$$

This holds naturally also for MP based frequency estimates, although the $\Delta[f]$ is determined in that case as the frequency increment of the code vectors.
4. Conclusion

In the present work, classical DFT and relatively new Matching Pursuit (MP) approaches were applied and compared in spindle frequency determination. There are of course many other methods too, but these two were selected because they are widely used and important. The outcome of the overall estimated spindle frequencies were in line with previous studies as most central spindles were centered near 13 Hz also in other studies [4,5,6]. There were some differences in the frequency values between the DFT and MP approaches, which was quite expected as the two methods are somewhat different and one operates in frequency domain and one in time domain. It is naturally a pity that the true frequencies of spindles are not known and the spindle frequency estimates could not be compared to those.

MP approach seemed to work well and the MP frequency resolution can be selected as high as desired with the expense of more code vectors and hence more computational load (far exceeding that of DFT). In DFT analysis, the fundamental frequency resolution (no zero padding used) of a 0.5 s long signal segment is as low as \( \Delta f = 2 \) Hz (\( f_c = 200 \) Hz) in the general case. That shows how accurately a certain frequency component can be resolved. In the present work, DFT and MP methods were applied also to shed light to how dominant the real-life spindle peaks are and whether zero padding can be used to improve the DFT based spindle frequency determination. The number of subjects was four, which is good to keep in mind, of course. Based on the outcomes of \( P_s \) and \( G_s \) analyses, it seemed that a clear majority of spindles (about 90%) were such that there was one clear spectral peak indicating one dominant spindle activity. This seems to indicate that in a clear majority of spindles, a more accurate estimate of spindle frequency can be obtained with FFT and zero padding than with fundamental DFT (no zero padding), which is quite interesting.

The outcome of the present \( P_s \) and \( G_s \) analyses is supported by the view that 5% of spindles seem to be superimposed spindles with multiple spindle activities [6]. One useful idea related to the measures \( P_s \) and \( G_s \) is that if they show low values indicating a superimposed spindle, then MP is likely a better choice than FFT or perhaps other methods could be applied [12].

Band-pass filtering (10.5-16 Hz) was necessary in both approaches (512-point FFT and MP) to allow a reasonable comparison between the methods. In additional testing with no filtering in FFT approach, there were many cases such that there was high sleep EEG activity just below 11 Hz and the FFT indicated peaks that were not present in the band-passed signal that MP approach used. The upper part of the spindle band behaved quite similarly, though, as expected.

Currently it would seem that it is quite sufficient for clinical and sleep research needs if the accuracy (\( \Delta f / 2 \)) of spindle frequency determination is in the order of 0.05 Hz. It seems that this level of accuracy can be reached with FFT and zero padding in as many as about 90% of spindles and with MP approach possibly in even more of them. Such accuracy can be used to quantify spindle frequencies very well.

References