SIGNAL TIME-FREQUENCY ANALYSIS FOR RF EMITTER DETECTION AND LOCATION

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ABSTRACT
The detection and location of Radio Frequency (RF) emitters is a very important capability for Electronic Surveillance (ES) of adversarial RF emissions for national security and/or defence purposes. Many detection and location techniques have been developed for various applications. This paper will detail algorithm developments and numerical analysis for detection and location of uncooperative RF emissions using two collaborating but spatially separated digital ES sensors. This paper summarizes the development, formalisms and implementations of Complex Cross-Ambiguity Functions (CAFs) to estimate time- and frequency- differences of signals collected by the two sensors. Since the received RF signal power is a potential challenge in many modern ES operational scenarios, the techniques developed depend heavily on compensating with long Coherent Processing Intervals (CPIs). This paper characterizes the method using numerical simulations in a representative scenario to demonstrate analytical results.

KEY WORDS
Cross-ambiguity function (CAF), sensors, RF emitters, location, TDOA and FDOA.

1. Introduction

Electronic Surveillance (ES) plays a critical role in electronic warfare as it includes threat detection, identification and localization. Such information is critically important in support of immediate decisions relevant to electronic attack and electronic protection to achieve advantages in the electronic warfare battlefield. Various detection and location techniques have been investigated in the past for different applications [1–4].

ES sensors are typically designed to detect and estimate Lines-of-Bearing (LOBs) from sensor sites to Radio Frequency (RF) transmission sites and, where possible, triangulate them to estimate the signal location. These methods normally require an array of at least two antennas connected to the sensor to perform Angle-of-Arrival (AOA) measurements before constructing approximate LOBs. In other cases, where AOA measurements are not available or are not suitable in terms of the level of location accuracy, other techniques can be used, such as the techniques based on estimates of time and frequency difference of signal arrival at the sensors [1,3–5].

Time-Difference-of-Arrival (TDOA) techniques from signals intercepted by separated sensors have been investigated in the past [1,3–6]. The resulting TDOA estimates can then be used to estimate the location of RF transmitters. The theoretical uncertainty bound of each TDOA estimate, known as the Cramer-Rao Lower Bound (CRLB), depends on the Signal-to-Noise Ratios (SNRs) in the receiving channels [1,6], and is inversely proportional to the Band-Width (BW) that the transmitted signal presents during the Coherent Processing Interval (CPI) under consideration. The detection of signals and the estimation of their TDOAs are typically implemented using a coherent correlation process. The maximum values produced by correlation processing are used to construct estimates of the TDOA. Each TDOA estimate can be used to generate an estimate of the corresponding Differential Range (DR) which, in combination with known locations of the two cooperating receiving platforms, can then be used to isolate the transmission to a hyperbolic surface in three-dimensional geometric space.

The location of RF emitting source can also be determined by estimation of the Frequency-Difference-of-Arrival (FDOA) between two or more receiving sensors [5,7,8]. The frequency difference is due to the relative motion between the receiving assets, resulting in frequency shifts from the original frequency of the emitting source. The CRLB of the FDOA depends on the SNRs in the received signals, and is inversely proportional to the integration time of the CPI. The estimates of the frequency shift can then be used to estimate the location of the emitting source.

Alternatively, location methods based on combining both TDOA and FDOA estimates from signals intercepted at multiple sensors may achieve location estimates that are superior to the results generated from using either TDOA or FDOA alone. Various algorithms have been developed and presented in the open literature for digitally and jointly estimating TDOA and FDOA, and using those joint estimates to perform geolocation or target tracking. The joint estimation is typically done through the construction of a complex Cross-Ambiguity Function (CAF). Each CAF is normally computed through some implementation of a generalized correlation between recordings of the same signal(s) collected by two separated sensors, each with independent sources of degradation such as additive thermal...
noise and/or phase noise. In [9], the CAF was considered as the natural generalization of the cross correlation; this has been followed by numerous other studies on the subject, including [10–14]. Methods for tracking uncooperative non-stationary transmitters using joint estimates of TDOA and FDOA have also been investigated in numerous studies, including [5, 7, 8].

As the future RF electronic warfare battlefield becomes more complex and congested with increasing numbers of difficult radar emitters (such as those with low probability of interception), the received signal power is a potential challenge in many operational scenarios that attempt to exploit this concept. Joint TDOA/FDOA estimators that operate in realistic environments may depend heavily on compensating for low signal levels with extended CPIs. The purpose of this paper is to investigate and advance a new technique for jointly detecting and locating RF emitters which might be difficult to detect and locate due to their low probability of intercepts. The paper provides details on developed algorithms and characterizes the method using numerical simulations in a representative scenario with varying levels of SNR.

2. Algorithm development

2.1 Concept

The generalized application concept is illustrated in Figure 1, where a collaborative system of two (or more) moving and/or stationary sensor monitors a pre-determined geospatial Region of Interest (ROI) that may contain uncooperative RF transmissions. The figure illustrates the relative position between the sensors and RF transmitters in the ROI.

2.2 Transmitted RF signals

Consider one of the RF emitters in the ROI with unknown position vector $\mathbf{x}_\rho$ in some Cartesian geospatial coordinate system, and radiating a modulated RF signal as given by

$$s(t) = A(t)e^{j2\pi f_0 t},$$

where $f_0$ represents the RF carrier frequency of the transmitter, and $A(t)$ is an unknown complex Base-Band (BB) representation of the signal including such effects as intrapulse modulation, and pulse envelopes.

2.3 Received Base-Band signal

Suppose that the location of each ES sensor can be described by a known time $t$ dependent Cartesian position vector $\mathbf{x}_\rho(t)$, where $\rho = 1, 2$ represents a sensor index. The range between the sensor $\rho$ and any point $\mathbf{x}$ in the ROI can be denoted by $r_\rho(\mathbf{x}; t) = \|\mathbf{x}_\rho(t) - \mathbf{x}\|$ at time $t$.

The received RF signal is related to the transmitted RF signal in equation (1) by various effects that include (but are not limited to) propagation delay caused by the range $r_\rho(\mathbf{x}; t)$, Doppler caused by the rate of range change with respect to the time $t$, and the effects of dispersion denoted below by a simplified complex scalar. (Other effects, such as the radar scan and RF multipath are not considered here.)

Figure 2 shows some basic steps that most modern digital RF ES systems might implement in order to generate BB signals from the received RF signals. The received RF signals are converted through a Down-Converter (DC) by using an analog Local Oscillator (LO) and then digitized with an Analog-to-Digital Converter (ADC). Thereafter, the digital signal can be converted to a digital complex BB representation using suitably designed digital down conversion and filtering. The aggregated frequency of the analog LO and digital DC is denoted here by $f_{LO}$.

Accounting for these effects, and invoking a linear approximation for $r_\rho(\mathbf{x}; t) = \|\mathbf{x}_\rho(t) - \mathbf{x}\|$ near some representative time $t \approx t_0$, the received BB signal is approximately equivalent to a digital representation of an analog signal given by

$$x_\rho(t) \approx G_\rho \exp(j2\pi f_\rho(\mathbf{x}_\rho; t_0)t) A(t - \tau_\rho(\mathbf{x}_\rho; t_0)),$$

where, for any geospatial position vector $\mathbf{x}$ in the ROI, at any time $t$, and sensor index $\rho$, the propagation delay and the Doppler-shifted frequency are respectively defined by

$$\tau_\rho(\mathbf{x}; t) = \frac{r_\rho(\mathbf{x}; t)}{c},$$

$$f_\rho(\mathbf{x}; t) = f_0 - f_{LO} - f_0 \frac{\tau_\rho(\mathbf{x}; t)}{c},$$

while $c$ represents the speed of light. (Explicit digital representations of such signals are not included here in the interest of notational simplicity.)
2.4 Joint estimation of time-frequency difference of arrivals

A proposed emitter detection and location is based on joint estimates of time and frequency difference arrival at the two sensors (shown in Figure 1).

TDOA estimation is the process of estimating the relative time delays between the same signal intercepted by the two separated sensors.

FDOA estimation is the process of estimating the relative frequency shift from the original frequency of the emitting source due to the relative motion between the sensors and/or transmitters.

For any geospatial position vector \( \mathbf{x} \) in the ROI and any time \( t \), one can define a line-of-sight TDOA and FDOA, respectively, by

\[
\tau(\mathbf{x}; t) = \tau_1(\mathbf{x}; t) - \tau_2(\mathbf{x}; t) = \frac{r_1(\mathbf{x}; t) - r_2(\mathbf{x}; t)}{c},
\]

(4)

and

\[
f(\mathbf{x}; t) = f_1(\mathbf{x}; t) - f_2(\mathbf{x}; t) = \frac{\dot{r}_1(\mathbf{x}; t) - \dot{r}_2(\mathbf{x}; t)}{-c/f_0}.
\]

(5)

TDOA and FDOA values can be estimated jointly at specific times by evaluating a complex CAF between digital representations of the two BB signals \( x_1(t) \) and \( x_2(t) \), received by sensor 1 and 2 respectively, and given by (2).

The analog representation of the CAF is given by

\[
A_I(\tau, f) = \int_I x_1(t) \overline{x_2(t + \tau)} \exp(-j2\pi ft) dt,
\]

(6)

where the integral operation is performed over a finite contiguous time interval, \( I \), known here as the CPI. The digital implementations of the CAF can be developed but are not presented here for the sake of of notational simplicity. One approach can be found in [9].

In practice, the linear approximation that allows the effect of Doppler to be parameterized by a single parameter \( \dot{r}_p(\mathbf{x}_e; t) \) in (2) and (3) does not apply for arbitrarily long time intervals since either range, \( r_1(\mathbf{x}_e; t) \) and/or \( r_2(\mathbf{x}_e; t) \), may depend non-linearly on time \( t \). It can be shown that the approximation leads to an upper bound on the duration of the CPI approximately given by

\[
\min \sqrt{\frac{2c}{f_0|\dot{r}_1(\mathbf{x}; t_0) - \dot{r}_2(\mathbf{x}; t_0)|}},
\]

(7)

where the minimization is evaluated over all \( \mathbf{x} \) in the ROI. Any CAF calculated over a CPI whose duration exceeds this bound may exhibit loss of resolution for some transmitter locations \( \mathbf{x} \) in the ROI.

2.5 Detection and location estimation

The joint TDOA/FDOA estimates are then determined and used to detect and locate any RF transmitters in the ROI.

It is straightforward to show in Equation (2) that, in the absence of noise or other sources of degradation and neglecting nonlinearity in the Doppler effects and other approximations, the maximum of \( |A_I(\tau, f)|^2 \) is expected to occur at the true values \( \tau(\mathbf{x}_e; t_0) \) and \( f(\mathbf{x}_e; t_0) \), for some representative time \( t_0 \) in the CPI.

In the presence of noise, joint estimates of \( \tau(\mathbf{x}_e; t_0) \) and \( f(\mathbf{x}_e; t_0) \) based on the CAF are denoted by

\[
(\hat{\tau}, \hat{f}) = \arg\max_{(\tau, f)} |A_I(\tau, f)|^2.
\]

(8)

The objective of the derivation of this technique is to use the estimates \( (\hat{\tau}, \hat{f}) \) from the CAF in order to form an estimate \( \hat{\mathbf{x}}_e \) of the true location \( \mathbf{x}_e \) of the emitter, simultaneously satisfying

\[
\tau(\mathbf{x}_e; t_0) = \hat{\tau},
\]

\[
f(\mathbf{x}_e; t_0) = \hat{f}.
\]

Equations (4) and (5) represent a mapping from some Cartesian coordinate system representing a three-dimensional geospatial volume \( \mathbb{R}^3 \) containing the ROI to a two-dimensional parameter space of TDOA/FDOA values \( \tau, f \) in \( \mathbb{R}^2 \). The mapping is both nonlinear and non-invertible. The mapping is normally invertible only if it is supplemented by at least one additional constraint, such as a two-dimensional topographical surface in \( \mathbb{R}^2 \) denoted here by

\[
g(\mathbf{x}) = 0.
\]

(9)

A unique estimate \( \hat{\mathbf{x}}_e \) can typically be obtained from unique values of \( \hat{\tau} \) and \( \hat{f} \) by solving the resulting non-linear system of three equations in \( \mathbb{R}^3 \). In some cases, such as when (9) represents a plane or a sphere and only one of the sensors is in motion so that (5) represents a cone, the solution can be constructed algebraically closed form. In other cases, it can be computed numerically.

Moreover, by computing and inverting the Jacobian matrix constructed from of gradients (derivatives with respect to the components of \( \mathbf{x} \)) of \( \tau(\mathbf{x}; t_0) \), \( f(\mathbf{x}; t_0) \) and \( g(\mathbf{x}) \), the Geometric Dilution of Precision can be quantified. This determines the overall conditioning of the geolocation problem including the sensitivity of uncertainties in location estimates to those of TDOA/FDOA joint estimates.

3. Numerical simulation

A set of simulations was executed to demonstrate the geolocation techniques using the CAF. The scenario used for simulation is illustrated by Figure 3 where only one stationary Signal Of Interest (SOI), (Tx), was located in the ROI at a point denoted below by \( \mathbf{x}_s \).

One ES sensor (Rx1) was ground-based and stationary. A second cooperating ES sensor (Rx2) was airborne and proceeded on a straight and level path with a constant horizontal velocity vector whose components were 5m/s
Table 1. Positions in geodetic [15] coordinates with respect to the WGS84 at start of simulation.

<table>
<thead>
<tr>
<th></th>
<th>Rx1</th>
<th>Rx2</th>
<th>Tx (x_e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lat.</td>
<td>45.347378N</td>
<td>45.352686N</td>
<td>45.353283N</td>
</tr>
<tr>
<td>Long.</td>
<td>75.887275W</td>
<td>75.952714W</td>
<td>75.892717W</td>
</tr>
<tr>
<td>Height</td>
<td>75 meters</td>
<td>884 meters</td>
<td>75 meters</td>
</tr>
</tbody>
</table>

east, 100m/s north and 0m/s up with respect to a horizontal tangent plane.

The locations of all of the assets at the starting time of the simulation, denoted below by T, are indicated in Table 1. All latitudes (“Lat.”) and longitudes (“Long.”) refer to horizontal geodetic coordinates of points on the 1984 World Geodetic System (WGS84) ellipsoid [15]. Points in three dimensions are specified by their vertical elevations (“Height”) directly above a point on the ellipsoid.

The SOI (Tx) was modelled to generate a periodic sequence of LFM rectangular pulses with a Pulse Repetition Interval (PRI) of 5 ms. The instantaneous frequency of each pulse increased linearly from 9315MHz to 9323MHz over a Pulse Duration (PD) of 1 ms.

The recording of digital BB data, representing the received signals x_1(·) and x_2(·) at sensors Rx1 and Rx2 respectively, was simulated by incorporating the effects of time delay and Doppler due to the time-dependence in the range between the sensors and the true location of the SOI. Each recording was randomized with additive white gaussian noise whose SNR (as determined by the squared signal amplitude and the noise power over a sensor bandwidth of 75 MHz) was −25 dB.

3.1 Representative CAF evaluations

Three representative CAFs, denoted by A_{I_1}(·; ·), A_{I_2}(·; ·), and A_{I_3}(·; ·), were evaluated numerically, each with a different CPI whose duration was 100 milliseconds. These CPIs represented the time intervals I_1 = [T + 3s, T + 3.1s], I_2 = [T + 13s, T + 13.1s] and I_3 = [T + 23s, T + 23.1s], respectively.

The locations of Rx2 at the mid-points of these three CPIs, denoted by t_1, t_2 and t_3 respectively, are represented by open blue circles in Figure 3. Figure 3 also includes typical “Isochron”, “Isodop” and planar “Ground” surfaces, under these conditions representing, respectively, the surfaces \( \tau(x; t_1) = \tau(x_e; t_1), f(x; t_1) = f(x_e; t_1) \), and \( g(x) = 0 \).

The evaluated CAF intensity \( |A_{I_1}(\tau, f)|^2 \) using data from the first CPI, I_1, is illustrated in Figure 4 over a finite rectangular region of TDOA and FDOA values, \{((\tau, f))\}. There, a green “X” represents the true TDOA and FDOA values \( \tau(x_e; t_1), f(x_e; t_1) \) of the SOI.

A geospatial rendering, or CAF-Map [14], of the CAF intensity in Figure 4 is illustrated in Figure 5. This represents the composition of the digitally evaluated CAF with the mapping in (4) and (5), \( |A_{I_1}(\tau(x; t_1), f(x; t_1))|^2 \). The composition is restricted to points \( x \) on the horizontal plane through \( x_e \) in a local east-north Cartesian coordinate system depicted in Figure 3. Note that the origin \( (0, 0) \) of this system has been arbitrarily chosen to represent the true location \( x_e \) of the SOI.

Figure 6 and Figure 7 represent similarly evaluated CAF-Maps, \( |A_{I_2}(\tau(x; t_2), f(x; t_2))|^2 \) and \( |A_{I_3}(\tau(x; t_3), f(x; t_3))|^2 \), over the remaining two CPIs. In order to compare these three CAF-Maps, an aggregated CAF-Map is presented in Figure 8 where local maxima from Figures 5, 6, and 7 appear in shades of red, blue and green respectively.

3.2 Monte Carlo simulations

In addition, a set of Monte Carlo (MC) simulations has been executed at various SNR levels in the above configuration to demonstrate the performance of the CAF to detect signals in noise. The degradation of the method in the presence of additive noise is important since many practical applications may require at least one of the two collaborating sensors to collect back-lobes or side-lobes where the SNR may be quite low.

For each simulation, one CAF was calculated over the CPI I_1. A “resolution ellipse” was drawn around the true values of the TDOA and FDOA values indicated in Figure 9 to represent the anticipated influence of the SOI on the CAF. The extent of the TDOA axis is the inverse of the signal bandwidth, \( 1/(8 MHz) = 0.125 $\mu$s. The ex-
Figure 5. CAF-Map around at $t_1 = T + 3.05s$.

Figure 6. CAF-Map around at $t_2 = T + 13.05s$.

Figure 7. CAF-Map around at $t_3 = T + 23.05s$.

Figure 8. Combined CAF-Map.

Figure 9. CAF maxima from CPI starting at $T + 3$.

The content of the FDOA axis is the inverse of the length of the CPI $1/(100\,ms) = 10\,Hz$. After each CAF evaluation, the maximum absolute CAF was identified on the region of FDOA values within $\pm 100\,Hz$ of the true value. The fraction of simulations for which this maximum occurred on the interior of the ellipse is plotted against the SNR for various selected values of the SNR indicated in Figure 9.

3.3 Discussions

Figure 4 indicates that the SOI was detectable using the CAF (and Figures 9 and 10 indicate that this is the most likely outcome) at the indicated SNR. However, ambiguities generally arise in the CAF when the SOI exhibits some degree of periodicity. In this case, the ambiguities are separated from one another by 200 Hz due to the PRI. Consequently, in the presence of noise, the highest local maximum is not always representative of the true TDOA and FDOA of the Signal of Interest (SOI).
These ambiguities may persist in the ROI of the CAF-Map as illustrated in Figures 5, 6 and 7. Consequently, their presence may preclude each individual CAF from providing the unique geospatial results anticipated by (8). However, as can be observed in Figure 8, if the SOI is stationary, then local CAF maxima near the true location appear to be corroborated by local maxima from multiple CPIs, whereas other local peaks are not.

This method can be used to detect transmissions with a certain level of SNR, and is expected to fail to detect signals with sufficiently low SNR as exemplified in Figure 10.

4. Conclusion

A joint TDOA/FDOA geolocation concept against uncooperative stationary radar sites has been described in this paper. This paper presents concept and techniques for detection and location of RF emitter transmissions. Numerical simulations have demonstrated the benefits of joint time-frequency estimation using CAFs for RF threat detection.

This paper contributes to an assessment of the ability of CAF methods to perform TDOA, FDOA and location estimation against difficult RF radar signals in modern warfare environments. Those radar signals tend to transmit very low power levels using complex waveforms. Consequently, they tend to be received only intermittently with very low SNRs. Under these conditions, other detection and location methods may not be effective. This paper also defines an upper bound on the duration of the CPIs for this specific application.

The simulations in this paper have specifically addressed estimate ambiguities related to waveform periodicities in one representative radar signal. Such ambiguities are inherent to all periodic waveforms. Potential future work in this area may include work on computationally efficient methods for discovering CAF peaks from radar signals, and methods to simultaneously track peaks and resolve ambiguities over sequences of CPIs.

In practice, for the application of CAF methods, cooperating receiving sensors require accurate clocks and LOs in order to accurately estimate time- and frequency-differences.

This research has potential applications against other transmissions. Many of these may not exhibit the pulsed periodicities or intercept intermittencies of typical radar signals. For example, communication signals are not usually periodic, exhibit much higher duty cycles and tend to be much less directional. Consequently, associated CAFs are expected to be much less ambiguous. However, the TDOA resolution is inversely proportional to the RF bandwidth.

References


