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SYSTEM RELIABILITY OF REINFORCED CONCRETE RIBBED ARCH BRIDGE

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Abstract

According to the mechanical characteristics of reinforced concrete ribbed arch bridges, the rib failure mode is proposed as the main failure mode of such bridges, and a mathematical model for reliability analysis of a reinforced concrete ribbed arch bridge system is established. The failure of such a system is studied by the method of critical strength with support constraints. By comparing various methods, the formal method of system reliability solution is determined to be the reliability calculation method of reinforced concrete arch bridge systems, and the calculation equation of the saliency coefficient of reinforced concrete ribbed arch bridge components is given. The process of reliability analysis of a reinforced concrete ribbed arch bridge system is summarized and its effectiveness verified by an example. The research results can provide references for the reliability analysis of reinforced concrete ribbed arch bridge systems.

Key Words

Reinforced concrete ribbed arch bridge, system reliability, importance coefficient, branch-and-bound method for critical strength, FORM method

1. Introduction

The reinforced concrete ribbed arch bridge has such advantages as a lightweight arch rib, simple bridge structure, low building cost, and cost-effective maintenance in the operational stage. A large number of reinforced concrete ribbed arch bridges have been built in the mountainous Southwest China regions to cross deep valleys (*e.g.*, Chongqing, Sichuan, Guangxi, Yunnan, and Guizhou) since the late 1980s up to the present. After many years of service, damage, such as cracked arch ribs, transversely placed deck slabs, and broken and damaged connections between arch ribs and spandrel columns, has occurred in the bridges, which affects their bearing capacity and service performance. The correct assessment of the bearing capacity of those bridges in service under economically and technically acceptable conditions is of importantly theoretical and practical significance.

Outside China, Moses [1] initially studied one classical multi-beam bridge in 1997 and found that the mere reliability index of the single main beam could determine the reliability index at the system level. Leng *et al.* [2] found in his study of the reliability of the superstructure of reinforced concrete bridges that the system reliability under ultimate limit state (ULS) was greater than the target reliability of key elements. Truong and Kim [3] integrated the improved Latin Hypercube (IHS), proposed effective importance sampling (EIS), and developed the IHS–EIS method for the reliability analysis of steel cable-stayed bridges. Kala [4] analysed the probability of failure of a load-bearing steel bridge member under bending and used Bayesian probability to study structural reliability, which identifies the optimal times for planning bridge inspections. Recently, many different methods for estimating the fatigue life of steel bridges have been proposed. Maljaars *et al.* [5] applied linear elastic fracture mechanics (LEFM) and finite element (FE) modelling to the analysis of cracks growing into the deck plates from the roots of the welds between the deck plates and stringers. Marques *et al.* [6] presented fatigue probabilistic analyses of steel-riveted bridges based on LEFM and FE modelling of the critical connections.

In China, Jiang and Xing [7] discussed the differences and connections between the reliabilities of the system and members of a typical bridge. Based on literature research and the existing achievements in this field, Li's research [8] showed that evaluation methods based on reliability theory have been used in practical engineering projects, but most were focused on assessing the safety of these bridges, and the means for structural system reliability assessment were relatively scarce. Liu *et al.* [9] considered the correlation among failures of all of the various elements of bridge engineering systems and provided an integral reliability analysis method for bridge systems. Analysing the advantages and disadvantages of the existing failure mode search methods and their applicability in bridge structure engineering, Gao and Ou [10] proposed an improved

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phase-critical-strength branch-and-bound method suitable for bridge structures, which solves the problem of searching the main failure modes of large bridge structures. Zhu *et al.* [11] utilized the Bayesian updating method to evaluate the system reliability of a cable-stayed bridge with inspection information. To make the computation more efficient, Liu *et al.* [12] developed a machine learning approach for system reliability evaluation of complex structures and applied it to cable-stayed bridges. Lu *et al.* [13] presented a framework for system reliability evaluation of in-service cable-stayed bridges subjected to cable degradation. The effect of cable strength degradation on the system reliability is demonstrated through a simulation on a parallel–series system representation.

All of the existing achievements of the study have shown that elements of the available current reliability theories cannot effectively assess the integral safety level of bridge systems. In this article, a reinforced concrete ribbed arch bridge was taken as the main object of study, and the reliability analysis method of such a bridge was explored.

2. Mathematical Model for System Reliability Analysis

In reinforced concrete ribbed arch bridges, their core structure comprises arch ribs and spandrels that transmit forces and loads. Once a failure of the arch rib as the core bearing system occurs, the entire bridge will collapse. As the main arch ribs and spandrels cannot jointly contribute to form a bearing structure, the statically indeterminate times that a reinforced concrete ribbed arch bridge fails are few compared with that of rigid arch and truss arch bridges. Thus, the failure mode of an arch rib is taken as the main failure mode for reinforced concrete ribbed arch bridges without considering the impacts of failures of other members on the system reliability of such bridges.

Hence, the system failure of a reinforced concrete ribbed arch can be presented in the form of the following events [14]:

$$\begin{cases} E_{fS} = \cup_{i=1}^{n} E_i \\ E_i = \cap_{k=1}^{q_i} E_i^k \end{cases} \tag{1}$$

where E_i represents failure occurring to the ith failure path and E_i^k that occurring to the kth member on the ith failure path. As mentioned earlier, the issue in the calculation of the system failure probability for a reinforced concrete ribbed arch bridge is one of working out a serial–parallel system failure probability.

3. System Reliability Analytical Method

3.1 System Failure Mode Exploration Method

The staged critical strength branch-and-bound method was used as the failure mode research method for reinforced concrete ribbed arch bridges, with the main procedure depicted by the flowchart shown in Fig. 1. First, the bridge system extreme state is defined and an FE model of bridge structure is established to carry out the calculation. Second, the calculation results are entered into the failure

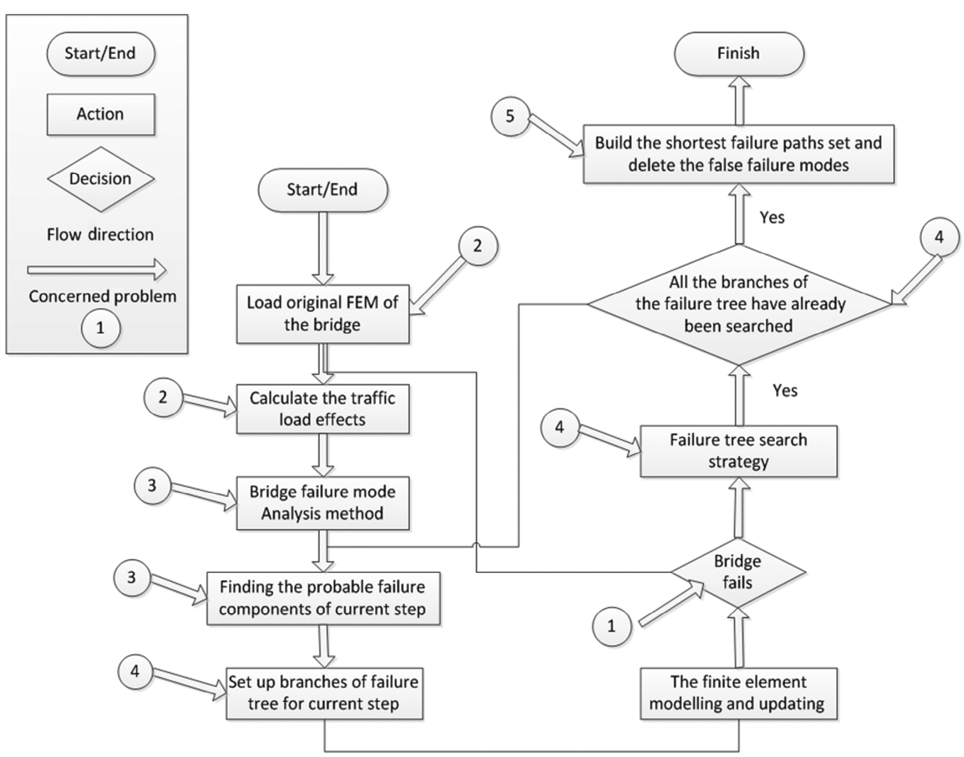

Figure 1. Flowchart of bridge structural failure mode research.

path search code to search failure members of the current stage and the failure member data are fed back to the FE procedure to modify the model topological structure. Then, the two previous calculations are repeated until the system failure occurs to form the first failure path and its branch of the system. Then, the information of the failure path is entered into the failure tree traversal research code to research new failure paths. Finally, all of the previous steps are repeated until the research for failure paths is complete to form a failure path set.

The calculation flow depicted in Fig. 1 shows that the failure mode searching for a reinforced concrete ribbed arch bridge mainly comprises the following key components [15]: (1) definition of the ultimate system state, (2) selection of the failure path searching code, (3) FE modelling for ribbed arches and achievement of topological structure variation, and (4) failure tree traverse research method.

3.1.1 Definition of Ultimate System State

The definition of ultimate system state is the definition of the basis for judging system failure. The failures of a reinforced concrete ribbed arch bridge can be grouped into the following three categories [15].

1. The number of failure elements of arch ribs reaches a certain fixed value m and the arch rib becomes a mechanism.
2. When the number of failure elements of arch ribs reaches a certain fixed value n, according to experience or specification requirement, considering the excessively large deformation, the structure has been considered to be unsuitable for bearing further additional loads.
3. The arch rib cannot bear additional load and ribbed arch bridge failure path searching code is required.

3.1.2 Failure Path Searching Code for Ribbed Arch Bridge

As the staged critical strength branch-and-bound method overall considers the failure state, failure evolution, and development history of the structural system, effective control in each stage of failure duration can be imposed on the failure tree branch. The structural unit strength and external loads on the structure can comprehensively reflect the structural strength and uncertainties in the stages of the structural analysis process. Thus, numerous computations can be reduced. Furthermore, this type of analytical method can utilize existing available structural analysis procedures and inherit the features of conciseness and high speed [16]. For these advantages, the staged critical strength branch-and-bound method is used as a system failure searching method for reinforced concrete ribbed arch bridges.

For the reinforced concrete ribbed arch bridge structure, vehicular load is considered as the main variable load, and hence vehicular load is taken as loads, and the corresponding calculation of staged critical strength branch-and-bound method is expressed as

$$\begin{cases} I_{r_k} = \mathrm{sign}[a_{r_k}^{(k)}] \\ R_{r_k}^{(k)} = R_{r_k}^{I_{r_k}} - I_{r_k} \times \sum_{i=1}^{k-1} a_{r_k}^{(i)} \Delta F_{r_i}^{(i)} m_{r_i} \\ \qquad = T_{r_k}^{I_{r_k}} - G_{r_k} - I_{r_k} \times \sum_{i=1}^{k-1} a_{r_k}^{(i)} \Delta F_{r_i}^{(i)} m_{r_i} \\ R_{S_{(r_k)}}^{(k)} = \Delta F_{r_k}^{(k)} + \sum_{i=1}^{k-1} \Delta F_{r_i}^{(i)} m_{r_i} = \sum_{i=1}^{k} (L_i \cdot R_{r_i}^{I_{r_i}}) \\ \Delta F_{r_i}^{(i)} = c_{r_i}^{(i)} q_k \end{cases} \tag{2}$$

where I_{r_k} is a sign function, $R_{r_k}^{(k)}$ is the residue bearing capacity of the member, $R_{S(r_k)}^{(k)}$ is the maximum external load in the kth stage of the failure path, along failure path $r_1 \to r_2 \to \cdots \to r_k$ that the system can bear, F_Q denotes the generalized load of the vehicular load, $\Delta F_{r_i}^{(i)}$ is the load increment imposed in ith stage, and $c_{r_i}^{(i)}$ is the load magnification times of the corresponding member r_i in the ith stage.

3.1.3 Establishment of Finite Element Model and Achievement of Topological Structure

The ultimate state reached by a reinforced concrete ribbed arch bridge is the process of progressive failure of all of the members of the arch ribs that finally causes the failure of the entire bridge. The FE model of a ribbed arch bridge must be renewed from time to time with an increment of the failed members. The failure data of members, such as the arranged number of the failure member, and failure type, are entered into the current FE model, and the FE model shall be modified and corrected based on the failure data entered. When the staged critical strength branch-and-bound method is applied as the failure path searching method, the finite modelling and modification can be made by using existing current structural FE analytical procedures (such as ANSYS and ABAQUS). The relevant information on all of the various failure members in various failure stages can be obtained directly using the staged critical strength branch-and-bound method.

3.1.4 Failure Tree Traversal Searching Method

After one failure path search is completed, how to start searching the next failure path and which failure path to be searched are important questions affecting whether the failure traversal can be completed rapidly without missing a failure path.

Seen from the point of view of data structure, the failure tree falls into a tree-shaped storage structure whose spread can be searched through an available computer programme. The first sequence traversal forest method of the forest traversal method under the data forest traversal method in data structure theory is applied in this article.

The following data structure describes the first sequence traversal forest method in detail.

1. Visit the root node point of the first tree in the forest.
2. Initially traverse the sub forest of the first tree.
3. In the elimination of the initial traverse, the forest is composed of the residual trees after the removal of the first tree.

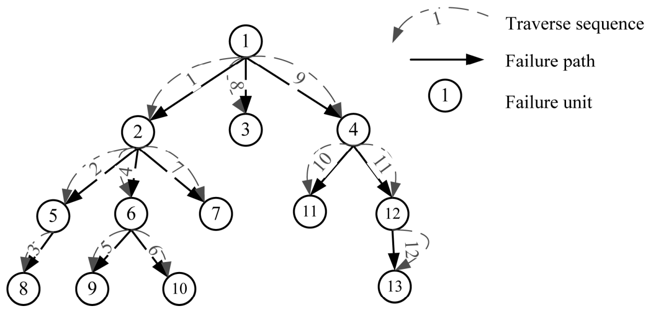

Figure 2. Presentation of the failure path and its searching process.

In the above-mentioned structure, the first sequence traversal refers to initially visiting the root node point of each tree in the forest and then visiting the left-hand subtree and finally the right-hand subtree.

The previous description explains the first sequence traversal forest method in the data structure, and when the method is applied to failure path searching, the detailed searching process can be clearly expressed in a concise way, as shown in Fig. 2.

3.2 System Failure Mode Exploration Method

Among all of the various point estimating methods, the FORM (first-order reliability method) method for obtaining system reliability is the simplest in form and convenient for calculation. This method directly establishes a relation between system reliability and element reliability and is convenient for analysing the sensitivity of the system failure probability of a structure. Hence, the FORM method is recommended for calculating the system reliability of a reinforced concrete ribbed arch bridge.

3.2.1 Solving Method of Model Failure Probability Calculation

One failure path equals one failure mode, and from the mathematical model of system analysis of a reinforced concrete ribbed arch bridge, a failure mode equals a parallel connected system, and the FORM method is used for calculating the parallel system failure probability. The calculation equation is:

$$p_{f,\text{mode}(i)} = \Phi_{m^{(i)}}(-\beta^{(i)}, R^{(i)}) \tag{3}$$

where $\beta^{(i)} = [\beta_1^{(i)}, \beta_2^{(i)}, \ldots, \beta_m^{(i)}]^T$ are the reliability indexes of all of the various failure elements in the ith failure path, $R^{(i)} = [\rho_{ij}^{(i)}]$ is a matrix of relevant coefficients of all of the various failure elements, and $m^{(i)}$ is the number of failure elements in the ith failure paths.

3.2.2 Resolving Method for Calculating System Failure Probability

The system failure of a reinforced concrete ribbed arch bridge equals the failure of parallel–serial systems composed of various failure modes, and the resolving method can be the FORM method for calculating the parallel–serial system failure probability. The calculation equation is

$$p_{f,Sys} = 1 - \Phi_m(\beta, R) \tag{4}$$

where $\beta = [\beta^{(1)}, \beta^{(2)}, \ldots, \beta^{(n)}]^T$ is the reliability index of all of the various failure paths for a reinforced concrete ribbed arch bridge, $R = [\rho_{ij}]$ is a matrix of related coefficients between the failure paths, and n is the total number of failure paths.

3.3 Importance Coefficient of Components

The reliability of all of the components of a reinforced concrete arch bridge is closely related to the system reliability of the entire bridge. Owing to different locations of components, different load sizes borne, and a bridge's different self-resistances, the components make different contributions to system reliability. To use the limited capital for bridge strengthening more rationally in the maintaining and strengthening courses of a reinforced concrete ribbed arch bridge, a purposeful strengthening strategy shall be stipulated by the degree of importance of different components; that is, the importance coefficient of the components of a reinforced concrete ribbed arch bridge is required. From the results of a system failure probability sensitivity analysis for a reinforced concrete ribbed arch bridge, the equation for computing the importance coefficient of the components of a reinforced concrete ribbed arch bridge is as follows:

$$\alpha_i = \frac{|\partial p_{f,Sys}/\partial \beta_{Elem,i}|}{\sum_{i=1}^{n} |\partial p_{f,Sys}/\partial \beta_{Elem,i}|} \tag{5}$$

where $\partial p_{f,Sys}/\partial \beta_{Elem,i}$ is the analysis result of the reliability sensitivity of the ith component by failure possibility.

The system reliability analysis flow procedure for a reinforced concrete ribbed arch bridge is concluded here and includes the following five aspects, and the detailed flow procedure is shown in Table 1.

3.4 Calculation Case

The reinforced concrete ribbed arch is taken as an object of study and its system reliability analysis is performed.

1. Searching mode for system failure

   The staged critical strength branch-and-bound method is used for searching the major failure mode of the specific ribbed arch bridge, and the bound parameter values $c_1 = 1.2$ and $c_i = 1 (i \neq 1)$ are taken. The major failure path of a given bridge is obtained through calculation.

   Failure Path 1: Node point 2 of unit 1→node point 1 of unit 1→node point 4 of unit 3→node point 5 of unit 5 (Fig. 3).

Table 1
Full Flow Procedure for System Reliability Analysis of a Reinforced Concrete Ribbed Arch Bridge

Number	Procedures	Contents
1	Determine the fundamental parameters for the main components at service based on inspection data and design data	Geometry of an arch rib section and reinforcement Concrete number Traffic volume increases rate Residual service period Vehicle operation state (average, dense) Standard value of the loading effects
2	Calculate reliability of the components at service	Set up resistance against decay model for the components Set up equivalent resistance probability model for components within the residual service duration Set up dead load effect probability model at service Set up vehicular load effect probability model at service
3	Search for the system failure model	Define system ultimate state Provide code for search failure path Set up FE model Provide failure tree traversal searching method
4	Calculate system reliability	Calculate the correlation coefficient between the failure components Calculate the correlation coefficient between the failure paths Calculate the reliability of all of the failure paths Calculate the system reliability
5	Calculate the importance coefficient of the components	Calculate the sensitivity of the failure paths Calculate the sensitivity of the failure paths to failure components Calculate the component importance coefficient

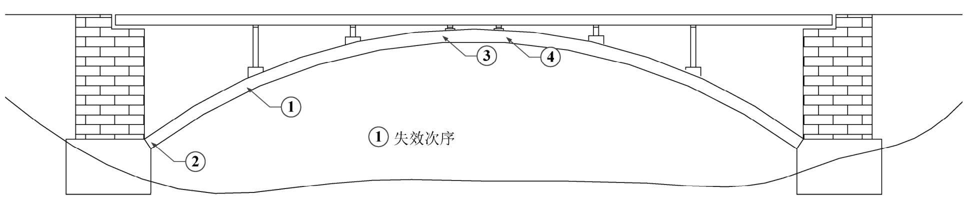

Figure 3. Failure path 1.

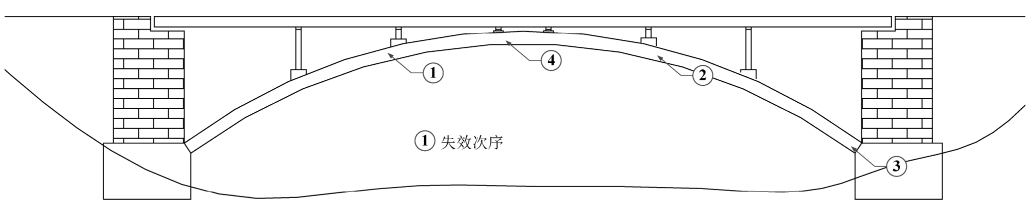

Figure 4. Failure path 2.

Failure path 2: Node point 3 of unit 2→node point 6 of unit 6→node point 8 of unit 7→node point 4 of unit 4 (Fig. 4).

2. Calculation of system failure probability

Equation (3) is used to calculate the failure probability of the various failure modes within various assessment periods, and (4) is used to calculate the system failure probability. The calculation results are shown in Table 2.

Table 2
System Reliability Analysis Results of Reinforced Concrete Ribbed Arch Bridge

Expected residual service period (years)	Design	50	30	10
System reliability index	5.66	4.31	5.00	5.58

Table 3
Importance Coefficients of Various Failure Components

Unit	Design	50	30	10
Node Point 2 of Unit 1	0.5625	0.4967	0.5190	0.5621
Node Point 1 of Unit 1	0.0000	0.0000	0.0000	0.0000
Node Point 4 of Unit 3	0.0000	0.0000	0.0000	0.0000
Node Point 5 of Unit 5	0.0000	0.0000	0.0000	0.0000
Node Point 3 of Unit 2	0.0752	0.2502	0.2261	0.1437
Node Point 6 of Unit 6	0.0973	0.2519	0.2374	0.1690
Node Point 8 of Unit 7	0.0000	0.0000	0.0000	0.0000
Node Point 4 of Unit 4	0.2649	0.0012	0.0176	0.1251

3. Component importance coefficient calculation

   Equation (5) is used to calculate the importance coefficient of the failure components within the different expected residual service periods as shown in Table 3.

   From the previous analysis, it is clear that among the failure components the reliability of node point 2 of unit 1, node point 3 of unit 2, node point 6 of unit 6, and node point 4 of unit 4 exert a significant influence on the system reliability.

4. Conclusion

In this article, the main failure mode of a reinforced concrete ribbed arch bridge is proposed, and the process of reliability analysis of a reinforced concrete ribbed arch bridge system is summarized. The mathematical model for the reliability analysis of a reinforced concrete ribbed arch bridge system is established and verified by an example. The main conclusions of this study can be summarized as follows:

- The arch rib failure mode is selected to be the main failure mode of a reinforced concrete ribbed arch bridge according to the characteristics of force bearing of such bridges without considering the impact of the components other than the arch rib on the system reliability of a reinforced concrete ribbed arch bridge; in addition, a system reliability analysis mathematical model for a reinforced concrete ribbed arch bridge is provided.
- The branch-and-bound critical strength method is used as the method searching for the system failure of a reinforced concrete ribbed arch bridge. System ultimate definitions, failure mode searching code, the topological structural variation realization method, and the failure tree traverse searching method are introduced. The FORM system reliability solving method is taken as the system reliability calculation method for an enforced concrete ribbed arch bridge system. Finally, the calculation formulation for the component significance coefficient of a reinforced concrete ribbed arch bridge is provided based on the calculation results of the system failure probability sensitivity analysis for such bridges.
- The full flow procedure of system reliability analysis for a reinforced concrete ribbed arch bridge is provided and verified by calculation cases.

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