loading torque, the rotor starts to accelerate. The measurements were performed on a two-pole induction machine of the output of about 1.1 kW. Speed at steady state was around 975 rpm. The ability of the motor to accelerate proves that the magnetic flux excited by the zero-sequence component passes through the air gap and induces currents in the rotor winding. A consequence is the rise of the torque. The fact that the steady-state speed was about 1/3 of the synchronous speed shows connection between the zero-sequence component and the third space harmonic of the current layer, the magnetic field and flux in the yoke. The described experiment denies the validity of the equation.

$$u_{0S} = R_S i_{0S} + L_{\sigma S} \frac{di_{0S}}{dt} \tag{6}$$

used in the theory of general machine [5] for the zero-sequence component. According to the equation the zero-sequence component $\mathbf{i}_{S0}$ does not give rise to the flux passing through the air gap and voltage drops arise only on the stator resistance R_S and on the stator leakage inductance $L_{\sigma S}$. Connection between the zero-sequence component and the third space harmonic of the current layer does not follow from the Eq. (6) either. Therefore, the considered emergency operation have to be analyzed by another method.

3. PROBLEM SOLUTION

For analysis of the considered way of feeding at emergency state, the method of space vectors and symmetrical components of instantaneous values was chosen, see [6]. The space phasor, e.g. the ν-th harmonic current layer of a 3-phase stator, can be written as

$$\mathbf{i}_{N\nu S} = \kappa_{\nu S} N_S \left(i_{SA} + \mathbf{a}^{\nu} i_{SB} + \mathbf{a}^{2\nu} i_{SC} \right), \tag{7}$$

where N_S represents the number of conductors of one phase windings, and $\kappa_{\nu S}$ is the winding factor of the ν-th space harmonic wave. The value **a** respects the space shifts of the stator phase windings. As it follows from Eq. (7), the position of the space phasor determines position of the ν-th space wave in the complex plane perpendicular to the axis of the machine and the magnitude of the space phasor is proportional to the maximum of this space wave.

The symmetrical components of instantaneous values of the stator currents are defined as

$$\mathbf{i}_{1S} = \tfrac{1}{3}\left(i_{SA} + \mathbf{a} i_{SB} + \mathbf{a}^2 i_{SC} \right) \tag{8}$$

$$\mathbf{i}_{2S} = \tfrac{1}{3}\left(i_{SA} + \mathbf{a}^2 i_{SB} + \mathbf{a} i_{SC} \right) \tag{9}$$

$$i_{3S} = \tfrac{1}{3}\left(i_{SA} + i_{SB} + i_{SC} \right) \tag{10}$$

Due to the periodically repeating values of the term in the parentheses in Eq. (7), it is evident that the first symmetrical component $\mathbf{i}_{1S}$ corresponds to the group of harmonics of the orders $\nu = 2+3k$, the second component corresponds to the harmonic of the orders $\nu = 2+3k$ and the third component to the wave of the orders $\nu = 3k$, where k is a non-negative integer. Similarly, space phasors of other considered quantities may be introduced. In case of multi-phase windings with m phases (e.g. cage winding), m symmetrical components arise. These components are significant mainly for their connection with groups of space harmonics, see [6]. It is obvious that the first, second and third symmetrical components of the instantaneous values cannot be confused with the positive-, negative- and zero-sequence components of the phasors defined by the Eqs (3) to (5). The second component is complex conjugated with the first component, while the negative component represents a system of phasors whose magnitude, generally speaking, is different from the magnitude of the positive-sequence component phasors. In case of symmetrical three-phase system, the negative component does not arise while the second component rises in any case. In a real machine higher space harmonics assigned to this component arise as well.

The third component, according to the definition (10), is a real quantity analogous to the zero- sequence component. By analogy to i_{3S} , the components u_{0S} and i_{0S} in Eq. (6) are defined.

In most currently produced three-phase induction motors, it suffices to take into consideration only basic harmonics of the group of waves assigned to the first and third components, i.e. the first and third space waves. For the above-mentioned way of feeding by injected currents equations

$$\frac{d\mathbf{i}_{1R\lambda}}{dt} + \left(\frac{R_{1R}}{L_{1R}} - j\omega_m \right) \mathbf{i}_{1R\lambda} = \frac{L_{1h}}{L_{1R}} \left(j\omega_m \mathbf{i}_{1S} - \frac{d\mathbf{i}_{1S}}{dt} \right) \tag{11}$$

$$\frac{d\mathbf{i}_{3R\lambda}}{dt} + \left(\frac{R_{3R}}{L_{3R}} - 3j\omega_m \right) \mathbf{i}_{3R\lambda} = \frac{L_{3h}}{L_{3R}} \left(3j\omega_m i_{3S} - \frac{di_{3S}}{dt} \right) \tag{12}$$

were derived in [7]. The index λ in rotor-current components denotes that these components are rated to the effective number of stator conductors of the corresponding harmonic. Symbol L_{1h} represents the main inductance for the first harmonic. Stator resistance R_S, rotor resistance R_R and leakage inductances $L_{\sigma S}$ and $L_{\sigma R}$ are usually known. According to [7], the main inductance L_{3h} for the third harmonic, resistance R_{3R} and leakage inductance $L_{3\sigma R}$ can be estimated as

$$L_{3h} = \frac{2}{9} \frac{\kappa_{3S}^2}{\kappa_{1S}^2} L_{1h} \tag{13}$$

$$R_{3R} = 2 \frac{\kappa_{1R}^2 \kappa_{3S}^2}{\kappa_{1S}^2 \kappa_{3R}^2} R_{1R}, \qquad L_{3\sigma R} = 2 \frac{\kappa_{1R}^2 \kappa_{3S2}^2}{\kappa_{1S}^2 \kappa_{3R}^2} L_{1\sigma R} \tag{14}$$

Symbols κ_{1R} and κ_{3R} represent the winding factors of the rotor winding. After substitution of Eq. (1) into Eqs. (8) and (10), the first and the third components can be written

$$\mathbf{i}_{1S} = \tfrac{1}{6} I_S \mathbf{K}_{1P} e^{j\omega t} + \tfrac{1}{6} I_S \mathbf{K}_{1N} e^{-j\omega t} = \mathbf{i}_{1SP} + \mathbf{i}_{1SN} \tag{15}$$

$$i_{3S} = \tfrac{1}{6} I_S \mathbf{K}_{3P} e^{j\omega t} + \tfrac{1}{6} I_S \mathbf{K}_{3N} e^{-j\omega t} = \mathbf{i}_{3SP} + \mathbf{i}_{3SN} \tag{16}$$

where

$$\mathbf{K}_{1P} = e^{j2\pi/3}\left(1 + e^{j2\pi/3} e^{j\varphi}\right), \ \mathbf{K}_{1N} = e^{j2\pi/3}\left(1 + e^{j2\pi/3} e^{-j\varphi}\right) \tag{17}$$

$$\mathbf{K}_{3P} = 1 + e^{j\varphi}, \qquad \mathbf{K}_{3N} = 1 + e^{-j\varphi} \tag{18}$$

The quantity $\mathbf{i}_{1SP}$ is an analogy to the positive-sequence component. Similarly, $\mathbf{i}_{1SN}$ respects the influence of the negative-sequence component. Decomposition of the third component into $\mathbf{i}_{3P}$ and $\mathbf{i}_{3N}$ is an analogy to the decomposition of pulsating field of the main winding of a single-phase motor into two complex conjugated quantities.

The symmetrical components of rotor currents are obtained by solving Eqs. (11) and (12) after substitution for $\mathbf{i}_{1S}$ and i_{3S} from Eqs. (16) and (17). This solution results in relations describing rotor current components in steady state

$$\mathbf{i}_{1R\lambda} = j\mathbf{K}_{1RP}\mathbf{i}_{1SP} + j\mathbf{K}_{1RN}\mathbf{i}_{1SN} = \mathbf{i}_{1R\lambda P} + \mathbf{i}_{1R\lambda N} \qquad (19)$$

$$\mathbf{i}_{3R\lambda} = j\mathbf{K}_{3RP}\mathbf{i}_{3SP} + j\mathbf{K}_{3RN}\mathbf{i}_{3SN} = \mathbf{i}_{3R\lambda P} + \mathbf{i}_{3R\lambda N} \qquad (20)$$

where

$$\mathbf{K}_{1RP} = \frac{L_{1h}(\omega_m - \omega)}{R_{1R} - jL_{1R}(\omega_m - \omega)},\ \mathbf{K}_{1RN} = \frac{L_{1h}(\omega_m - \omega)}{R_{1R} - jL_{1R}(\omega_m + \omega)} \qquad (21)$$

$$\mathbf{K}_{3RP} = \frac{L_{3h}(3\omega_m - \omega)}{R_{3R} - jL_{3R}(3\omega_m - \omega)},\quad \mathbf{K}_{3RN} = \frac{L_{3h}(3\omega_m - \omega)}{R_{3R} - jL_{3R}(3\omega_m + \omega)} \qquad (22)$$

From the preceding equations it is obvious that magnitudes of the components of rotor currents are proportional to the magnitudes of the corresponding components of stator currents. Therefore, it is possible to reduce magnitudes of both negative and zero-sequence components by a convenient choice of the angle φ.

As it further follows from Eqs (20) and (22), as a reaction to the zero-sequence component, a corresponding component in rotor currents arises, which results in increase of stray losses in the rotor winding. These losses can be considerable. On the other hand, it can be supposed that the influence of the zero- and the negative-sequence components on the mean value of torque will be quite small, because the machine in the steady state operates with speed considerably different from the synchronous speed corresponding to the zero- and negative-sequence components. However, the rise of considerable torque ripples can be expected due to the interaction of positive- and negative-sequence components of stator and rotor currents [8].

4. REDUCTION OF STRAY LOSSES AND PARASITIC TORQUES

As it has been already mentioned in the preceding text, the driving torque is only produced by the positive-sequence component of the currents, while the influence of the other components on the mean value of the torque can be neglected. The positive-sequence component is respected by the quantity $\mathbf{i}_{1SP}$. The magnitude of $\mathbf{i}_{1SP}$ is according to (15) proportional to the absolute value of the constant $\mathbf{K}_{1P}$ defined by the Eq. (17). From Eqs. (15) and (17) it follows that $\mathbf{i}_{1SP}$ will reach its maximum when the angle choice is φ = -120°, when $|\mathbf{K}_{1P}| = 2$ and $|\mathbf{K}_{1N}| = 1$. If the machine is required to work with the nominal torque, the magnitude $\mathbf{i}_{1SP}$ must be equal to the magnitude $\mathbf{i}_{1S}$ at the standard operation before the rise of failure. For standard operation, $|\mathbf{i}_{1S}| = 1/2\, I_{Sn}$, where I_{Sn} is the amplitude of the nominal current. From Eqs. (15) and (17) $I_S = 3/2\, I_{Sn}$ follows for the amplitude of the stator current in emergency operation. In Fig. 3 waveforms of real α and imaginary β parts of the first component $\mathbf{i}_{1S}$ and the third component (zero-sequence component) i_{3S} of stator currents are shown. Calculations were performed for a machine with the main parameters in per unit system: $R_S = R_{1R} = 0.02$, $L_{1S} = L_{1R} = 9.55\times10^{-3}$, and $L_{1h} = 9.23\times10^{-3}$. Further stator winding with nine slots per pole pair and with step shortened by one slot pitch was considered. In this case, $\kappa_{1S} \cong 0.945$ and $\kappa_{3S} \cong 0.577$ hold true for the winding factors. Those of cage windings can be considered equal to one for simplification.

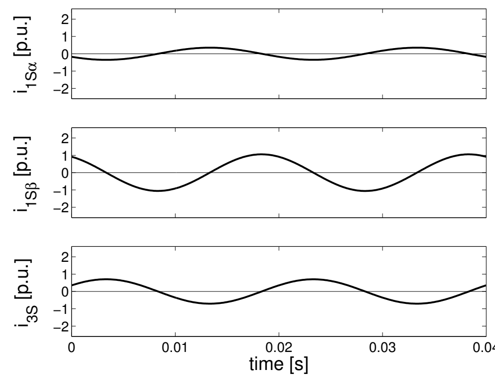

Fig. 3. Components of stator currents, φ = -120°

In Fig. 4 there are loci of space phasors $\mathbf{i}_{1S}$ and i_{3S}. The locus of phasor $\mathbf{i}_{1S}$ has the shape of ellipse due to the negative-sequence component. The locus of the phasor i_{3S} is an abscise laying on the real axis. Waveforms of real d and imaginary q parts of the first and the third components of rotor currents are depicted in Fig. 5 in rotor coordinates.

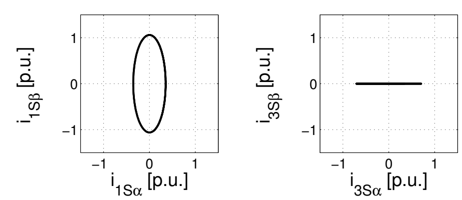

Fig. 4. Loci of the first and the third stator current components, φ = -120°

The torque for the first may be written as

$$T_1 = 6pL_{1h}\mathrm{Re}\left[j\mathbf{i}_{1S}^{*}\mathbf{i}_{1R\lambda}\right] \qquad (23)$$

The torque generated by the third component is

$$T_3 = 9pL_{3h}\mathrm{Re}\left[j\mathbf{i}_{3S}^{*}\mathbf{i}_{3R\lambda}\right] \qquad (24)$$

Analogous equations can be used for calculation of torque components. The total torque T is given by the sum of T_1 and T_3. Fig. 6 shows courses of torque T_{1P}, which is generated by the positive-sequence components, T_{1N} generated by the negative-sequence component and pulsation components of the torque T_{1PN}, which arises due to interaction between the positive-sequence component of stator current and the negative-sequence component of the rotor currents. The quantity T_{1NP} is arises due to the interaction of the negatives-sequence component of the stator currents and the positive-sequence component of the rotor currents. Braking torque T_{1N} may be neglected in comparison to the nominal value of torque generated by the positive-sequence component. Courses of the resulting torque T_1 generated by the first harmonic, torque T_3 generated by the third harmonic of currents and the total torque T are in Fig. 7. The torque oscillates with the amplitude about 1/2 of the nominal torque.

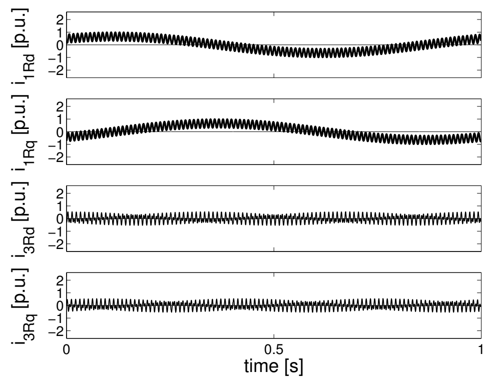

Fig. 5. Components of rotor currents, φ = -120°

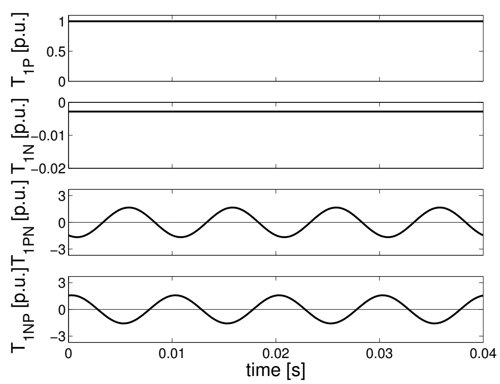

Fig.6. Torques T_{1P}, T_{1N}, T_{1PN}, and T_{1NP}, φ = -120°

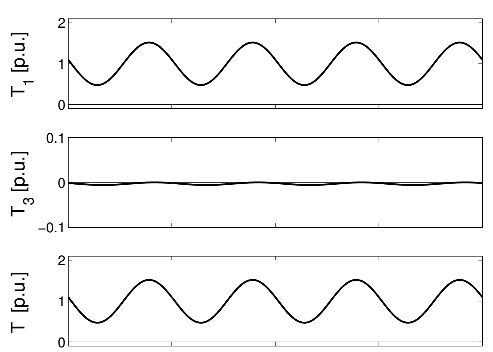

Fig. 7. Torques T_1, T_3, and T, φ = -120°

The influence of these ripples to the driven device will be considerably damped by the moment of inertia of the rotor [9]. The influence of torque generated by the third space waves may be omitted. It is evident that the mean value of the torque is given by the torque generated by the positive-sequence component. Its decrease by braking effect of the positive- and zero-sequence components is lesser than 1% of the nominal value. The loci of the space phasors of magnetic flux of the first and the third harmonics are shown in Fig. 8. Distortion of the locus of the first harmonic by the negative-sequence component is quite small. The third harmonic gives rise to pulsations of the flux. Amplitude of these pulsations is about 7% of the nominal value. So it can be presumed that the influence of the negative-sequence component on the magnetic circuit of the machine is quite small and, therefore, it can be neglected.

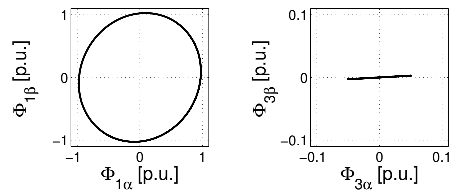

Fig. 8. Loci of the first and the third space phasor of flux, φ = -120°

Losses in stator winding in operation before the failure P_{Sn} and losses P_{Sf} after failure can be estimated as

$$P_{Sn} = 3R_S\left(\frac{1}{\sqrt{2}} I_{Sn}\right)^2, \quad P_{Sf} = 2R_S\left(\frac{3}{2\sqrt{2}} I_{Sn}\right)^2 \qquad (25)$$

The proportion $P_{Sf}/P_{Sn} = 1{,}5$. In comparison to nominal operation, losses in emergency operation increase by 50%. Similar increase of losses can also be expected in the rotor winding.

In [3], the angle φ = -60° is proposed for emergency operation. In a similar way as in the preceding case it can be derived that the amplitude of stator currents must be increased to $\sqrt{3} I_{Sn}$ in order that the motor can work with the nominal torque even after converter reconfiguration. Then $\mathbf{K}_{1N} = 0$ and negative component does not arise. The loci of space phasors of the stator currents are in Fig. 9.

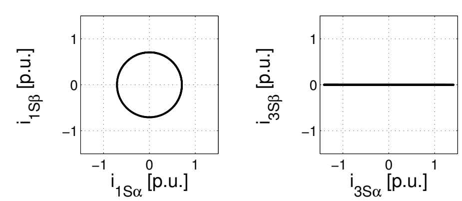

Fig. 9. Loci of the first and the third stator current components, φ = -60°

The locus of $\mathbf{i}_{1S}$ is a circle. Figure 10 shows courses of torque components T_1 and T_3 and the total torque. Torque T_1 is not distorted by the negative-sequence component. Torque generated by the third harmonic decreases the mean value of the total torque by about 1% of the nominal value. Losses P_{Sp} are

$$P_{Sp} = 2R_S\left(\frac{\sqrt{3}}{\sqrt{2}} I_{Sn}\right)^2 \qquad (26)$$

Proportion $P_{Sp}/P_{Sn} = 2$. The increase of stray losses is 100% compared to the standard operation. As it follows from (17), the zero-sequence component does not arise at $\varphi = 180°$. Amplitude of stator current must be chosen in the same way as in the preceding case. For that reason, the increase of stray losses will be the same. In this case, the machine operates similarly as a single-phase motor. As it follows from Fig. 11, pulsating torque T_{PC} arises with the amplitude approximately equal to the nominal torque. Decrease of the mean value of the torque is about 1% of the nominal value. The minimal stray losses arise at the choice $\varphi = -120°$. If, with regard to the converter or motor, it will be necessary to decrease the drive's output, the best choice for the minimal decrease of power will be $\varphi = -120°$, see [10].

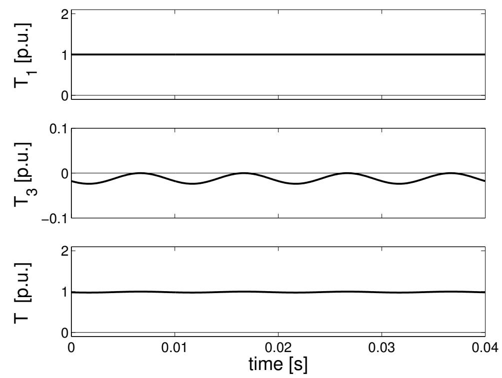

Fig. 10. Torques T_1, T_3, and T, $\varphi = -60°$

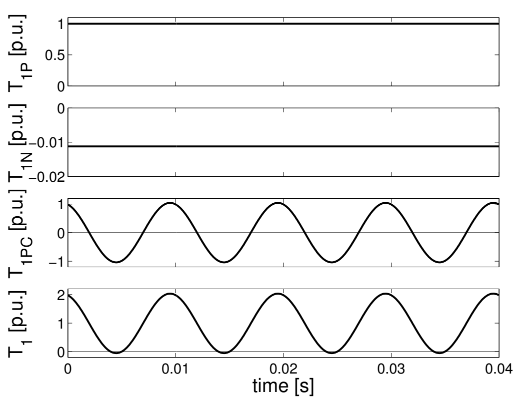

Fig. 11. Torques T_{1P}, T_{1N}, T_{1PC}, and T_1, $\varphi = -60°$

5. EXPERIMENTAL VERIFICATION

Experimental verification is based on the fact that the mean value is proportional to the amplitude of the positive-sequence component of stator currents represented by the quantity $\mathbf{i}_{1SP}$. As it can be derived from Eqs. (15) to (18), the magnitude $\mathbf{i}_{1SP}$ is only function of the angle φ at a given mechanical speed ω_m and at a given amplitude of stator currents I_S. As it follows from the theory of electrical machines, the torque in the air gap in steady state is proportional to square of the stator currents. It is evident that the proportion of torques for two different angles φ will be equal to the square of proportion of magnitudes $\mathbf{i}_{1SP}$ for these angles. For example, for $\varphi = -120°$ according to Eqs. (15) to (17) is $\left|\mathbf{i}_{1SP}\right|_{120} = 2/6 I_S$ and for $\varphi = -60°$ is $\left|\mathbf{i}_{1SP}\right|_{60} = \sqrt{3}/6 I_S$. The proportion of these values is $\sqrt{3}/2$. The proportion of torque values for these angles is 4/3. Therefore, the torque generated by the machine at $\varphi = -120°$ is 1.33 multiple of the torque generated for $\varphi = -60°$. For $\varphi = -120°$ and $\varphi = 180°$ proportion of torques will hold the same value.

The amplitude and phase shift of stator currents can be set by means of a circuit described in the scheme in Fig. 12. This circuit consists of a three-phase adjustable autotransformer TR1 and three adjustable transformers TR2 to TR4. A convenient setting of the output voltage of the transformers can provide such voltages at the stator phase windings B and C that currents of these windings will be of requested amplitude and phase shift φ. In Fig. 13 a, there is a three-phase system of voltage phasors feeding the input terminals of the circuit in Fig. 12.

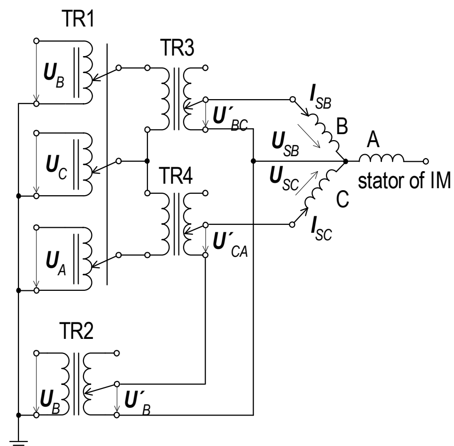

Fig. 12. Scheme for experimental measurement

The principle of stator voltage generation is shown in Fig. 13 b. Voltage phasor $\mathbf{U}_{SB}$ of the phase B is identical with phasor $\mathbf{U}'_{BC}$ of the secondary winding of transformer TR3. Voltage phasor $\mathbf{U}_{CA}$ of phase C is given by the sum of the voltage phasors $\mathbf{U}'_{CA}$ and $\mathbf{U}'_{B}$ of the secondary winding of transformers TR4 and TR2. In case, the angle between quantities $\mathbf{U}_{SB}$ and $\mathbf{U}_{SC}$ is bigger than 120°, terminals of transformer winding TR2 have to be interchanged.

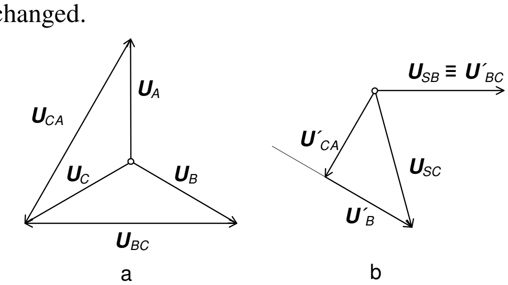

Fig. 13. Stator voltage generation

The four-pole motor of nominal output 1,5 kW was used for the measurements. The torque was measured by means of a dynamometer. The speed was kept at 1430 rpm. The amplitude of currents was 3.6 A. Fig. 14 shows stator voltages and currents for $\varphi = -120°$. Phase shift of voltages differs from that of currents. The measured value of torque was 2.6 Nm. Voltages and currents for $\varphi = -60°$

are in Fig. 15. The magnitude of torque was 2 Nm. Proportion of torques for φ = -120° and φ = -60° is 1.3. Voltages and currents for φ = -180° are in Fig. 16. Torque proportion for φ = -120° and φ = 180° is 1.25.

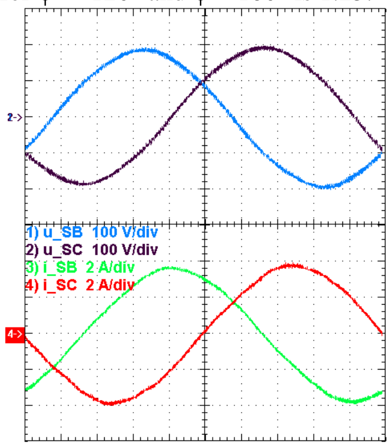

Fig. 14. Measured voltages and currents, φ = -120˚

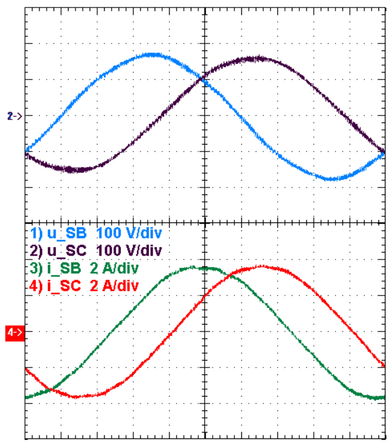

Fig. 15. Measured voltages and currents, φ = -60˚

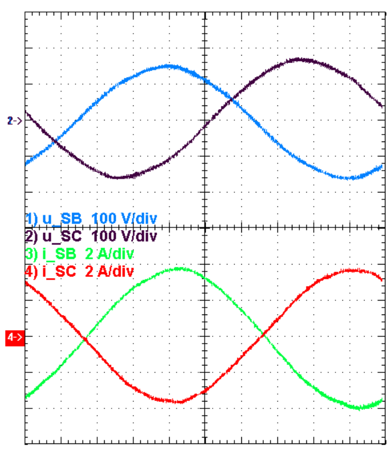

Fig. 16. Measured voltages and currents, φ = -180˚

6. CONCLUSION

The results of measurements are in accordance with the results of theoretical analysis. The measurements confirmed the fact that at emergency feeding by injected currents the optimal value of phase shift of the currents is -120°. At this value the machine operates with minimal stray losses. From the viewpoint of torque ripples, choice of the phase shift -60° seems to be advantageous. In this case, however, the stray losses are two times greater than the losses at phase shift -120°.

ACKNOWLEDGEMENT

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