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TWO BUS BAR POWER SYSTEM SHUNT COMPENSATION

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ABSTRACT
The paper presents two methods for determining the shunt compensation at the receiving end of a two bus bar system. The effect of shunt compensation on the power that can be transferred through a long transmission line is illustrated through the use of a two bus bar system. It is shown that for a long transmission line the voltage at the receiving end is higher than at the sending end. Also the power that can be transferred through a long line is much reduced due to large voltage drops. Shunt compensation is shown to maintain a constant voltage at the receiving end, for powers up to the maximum power that can be transferred through the line.

KEY WORDS
Modeling, shunt compensation, maximum power flow

1. Introduction

The paper presents two methods for determining the shunt compensation that is required to give a specified voltage at the receiving end substation of a two bus bar system. Then the methods are used to determine the shunt compensation that is required to improve the performance of a practical transmission line

A two bus bar system is useful for illustrating the effect of the length of transmission line on the performance of a transmission system. Transmission of power through a transmission line is accompanied by the consumption of lagging reactive power and generation of leading power in the reactance and susceptance of the line respectively. The length of a transmission line predominates in determining the amounts of reactive powers that are generated and consumed by a transmission line. This is because the voltage at which the line is operated at does not vary appreciably between no-load and full-load, due to statutory requirements that load voltages should in general be within +/- 10% of their nominal values.

The voltage at the load is also influenced by the power factor of the load. Lagging power factors result in lower receiving end voltages while leading power factors higher receiving end voltages; with reference to the voltages at unity power factor [1].

When the load voltages are outside the limits they may be controlled by introducing reactive shunt compensation. This may be either series or shunt compensation. In the latter capacitors are introduced in series with the line to raise the receiving end voltage whilst in the latter the compensation shunts the load, i.e. it parallels the load [2]. For a long transmission line the shunt compensation is inductive at low loads to hold the load voltage down and capacitive at high loads to raise the load voltage. Therefore the value of the compensation required to maintain the voltages at the desired voltage at the load bus bar varies with the load.

Shunt compensation is an established way of holding the load voltage constant, or within the operating limits, between low and high load conditions. Shunt compensation avoids the complex switching arrangements that series compensation requires to cater for faulty conditions, as well as the possibility of subsynchronous resonance (with generation) [3]. Furthermore shunt compensation improves the voltage stability of the transmission system as well as the maximum power that can be transferred to the load [4][5][6][7].

The shunt compensation that is required to meet the transmission conditions may be obtained by either a graphical solution or by an analytical approach. In the former method charts are drawn from which the compensation may be determined. Its accuracy is limited as is its application, especially when considering various loading conditions.

Conventional analytical methods tend to be based on simplifying assumptions, such as assuming a lossless line when determining the compensation required to hold the voltage at the end of the transmission line equal to the value at the sending end, or assuming that the reader can manipulate the power transfer equations to obtain the compensation required on load.

Two alternative methods are introduced in this paper for determining the compensation for different loadings from no load to the maximum permissible loading of a transmission line with losses. The methods are then applied on a practical line in the power system of Botswana.

The first method uses a spreadsheet to solve by successive iterations for the compensation that gives the specified voltages at the sending and receiving ends of the transmission line. In the second method an analytical solution is developed that gives the compensation to give the desired voltages at the two bus bars.

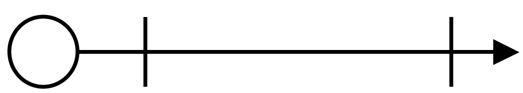
The analytical background of the proposed two methods is given in section 2 followed by results of voltage variations at the load end without shunt compensation for a medium length line and a long line in section 3. The results for a network with a long transmission line which has compensation are given in section 4. Conclusions are presented in section 5.

2. Analytical considerations

It is required to find the compensation that is needed to hold the voltages in the two bus bar system shown in Figure 1. The transmission network comprises a 132 kV transmission line rated 90 MVA. Two lengths of 150 km and 498 km correspond to some practical distances (Francistown to Orapa and Francistown to Maun respectively) on which the line has been used. At Maun the transmission line terminates in a substation with two 10 MVA transformers and a shunt reactor of 5 MVAr.

The parameters of the line are as follows: -

Resistance 0.1912 Ω/km (Given)
Reactance 0.3073 Ω/km (Given)
Susceptance 3.56825E-06 S/km (Estimated)
Thermal Rating 90 MVA



The two lengths of the line, 150 km and 498 km, are used to simulate a medium line and a long line respectively. In either case the line is represented by its nominal π representation. Compared to the calculated (exact) equivalent π representation the series arm impedance is higher by 4.5% while each of the shunt arms admittance is lower by 2.3%.for the length of 498 km. The corresponding errors for the 150 km line are much lower at 0.4% and 0.2% for the series and shunt arms respectively.

The voltage and voltamperes bases used in the analysis are 132 kV and 50 MVA respectively. The latter is 84.2% of the surge impedance loading of the line.

The performance of the line is determined for loads between no load and the maximum value that can hypothetically be transmitted through the line. Three power factors of 0.9 lagging, unity and 0.9 leading are used for the cases of no compensation and with compensation for the medium and long line configurations.

The voltage at the sending end is taken as the reference with its magnitude at 1.0 per unit and angle of 0˚.

Spreadsheet Solution

The active power P_1 and reactive power Q_1, for the power system in Figure 1, are given by the following equations: -

$$\begin{aligned} P_1 &= V_1^2 G_{11} + V_1 V_2 (G_{12}\cos\theta_{12} + B_{12}\sin\theta_{12}) \\ Q_1 &= -V_1^2 B_{11} + V_1 V_2 (G_{12}\sin\theta_{12} - B_{12}\cos\theta_{12}) \end{aligned} \quad \ldots(1)$$

Where $G_{ij} + j\,B_{ij}$ is the ij^{th} *element* of the bus admittance matrix

$P_1 + jQ_1$ is the load, inclusive of shunt compensation, at the load bus bar

V_i is the voltage magnitude at bus bar i

θ_{12} is the angle between bus bars 1 and 2.

Rearranging the equations by taking the terms with voltages to the left of the equals sign results in: -

$$\begin{aligned} \frac{P_1 - V_1^2 G_{11}}{V_1 V_2} &= G_{12}\cos\theta_{12} + B_{12}\sin\theta_{12} \\ \frac{Q_1 + V_1^2 B_{11}}{V_1 V_2} &= G_{12}\sin\theta_{12} - B_{12}\cos\theta_{12} \end{aligned} \quad \ldots(2)$$

Squaring both sides and adding: -

$$\left(\frac{P_1 - V_1^2 G_{11}}{V_1 V_2}\right)^2 + \left(\frac{Q_1 + V_1^2 B_{11}}{V_1 V_2}\right)^2 = \begin{bmatrix} (G_{12}\cos\theta_{12} + B_{12}\sin\theta_{12})^2 \\ + (G_{12}\sin\theta_{12} - B_{12}\cos\theta_{12})^2 \end{bmatrix}$$

Expanding the terms and rearranging results in: -

$$\frac{\left[V_1^4 (G_{11}^2 + B_{11}^2) + V_1^2 (2Q_1 B_{11} - 2P_1 G_{11}) + P_1^2 + Q_1^2\right]}{V_1^2 V_2^2} = G_{12}^2 + B_{12}^2$$

Which is a quadratic equation in V_1^2: -

$$\begin{bmatrix} V_1^4 (G_{11}^2 + B_{11}^2) + V_1^2 \begin{Bmatrix} 2(Q_1 B_{11} - P_1 G_{11}) \\ -V_2^2 (G_{12}^2 + B_{12}^2) \end{Bmatrix} \\ + P_1^2 + Q_1^2 \end{bmatrix} = 0$$

Whose solution is: -

$$V_1^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad . . . (3)$$

Where $a = G_{11}^2 + B_{11}^2$

$$b = \left\{2(Q_1 B_{11} - P_1 G_{11}) - V_2^2 (G_{12}^2 + B_{12}^2)\right\}$$

$$c = P_1^2 + Q_1^2$$

The angle of the voltage at the load bus bar is found from solving the equations (2) for θ_{12}.

$$\theta_{12} = \sin^{-1}\left[\frac{\begin{Bmatrix} B_{12}(P_1 - V_1^2 G_{11}) \\ + G_{12}(Q_1 + V_1^2 B_{11}) \end{Bmatrix}}{(G_{12}^2 + B_{12}^2) V_1 V^2}\right] . . . (4)$$

The maximum power that can be transferred through the line is given by the limiting condition in equation (3); i.e. $b^2 - 4ac = 0$. Substituting for the values leads to: -

$$\begin{bmatrix} \left\{2(Q_1 B_{11} - P_1 G_{11}) - V_2^2 (G_{12}^2 + B_{12}^2)\right\}^2 \\ -4\left(G_{11}^2 + B_{11}^2\right)\left(P_1^2 + Q_1^2\right) \end{bmatrix} = 0 . . . (5)$$

Denoting power factor as pf

$Q_1 = P_1 \tan(\cos^{-1}(pf))$ for lagging power factor and
$Q_1 = -P_1 \tan(\cos^{-1}(pf))$ for leading power factor.

Expanding equation (5) for lagging power factor results in the following quadratic equation in P_1^2: -

$$4P_1^2 \begin{bmatrix} -2G_{11}B_{11}\tan(\cos^{-1}(pf)) \\ -G_{11}^2 \tan^2(\cos^{-1}(pf)) - B_{11}^2 \end{bmatrix}$$

$$-P_1 \left[4V_2^2 V_2^2 \begin{Bmatrix} (G_{12}^2 + B_{12}^2) \\ (B_{11}\tan(\cos^{-1}(pf)) \\ -G_{11} \end{Bmatrix} \right]$$

$$+V_2^4 (G_{12}^2 + B_{12}^2)^2 = 0$$

Define for lagging power factor: -

$$a' = 4\begin{bmatrix} -2G_{11}B_{11}\tan(\cos^{-1}(pf)) \\ -G_{11}^2 \tan^2(\cos^{-1}(pf)) - B_{11}^2 \end{bmatrix}$$

$$b' = -4V_2^2 \begin{Bmatrix} V_2^2 (G_{12}^2 + B_{12}^2) \\ (B_{11}\tan(\cos^{-1}(pf)) - G_{11} \end{Bmatrix}$$

$$c' = V_2^4 (G_{12}^2 + B_{12}^2)^2$$

For leading power factor Q_1 is of opposite sign therefore the factors become: -

$$a' = 4\begin{Bmatrix} 2G_{11}B_{11}\tan(\cos^{-1}(pf)) \\ -G_{11}^2 \tan^2(\cos^{-1}(pf)) - B_{11}^2 \end{Bmatrix}$$

$$b' = \begin{Bmatrix} -4V_2^2 (V_2^2 (G_{12}^2 + B_{12}^2) \\ (-B_{11}\tan(\cos^{-1}(pf)) - G_{11}) \end{Bmatrix}$$

$$c' = V_2^4 (G_{12}^2 + B_{12}^2)^2$$

The maximum power is given by: -

$$P_{max} = \frac{-b' + \sqrt{b'^2 - 4a'c'}}{2a'} \quad . . .(6)$$

where the positive sign has been taken for the factor under the square root sign because for a load the value of P_{max} is negative.

The voltage magnitude and the angle of the voltage at the load bus bar are obtained from equations (3) and (4). They were solved on a Microsoft Office Excel spreadsheet. This gave the two values of voltages for a given load, with the exception that for the maximum power transfer condition the two voltages were equal. In the case where compensation was to be determined it was found by varying the compensation until the desired voltage at the load bus bar was obtained. The variation of the voltage magnitude with power for each power factor was then plotted and the results are presented in Sections 3 and 4 for the cases without compensation and with shunt compensation respectively.

Table 1 shows a spreadsheet table for a power factor of 0.9 lagging. $V_{lagupeer}$ and $V_{laglower}$ are the two voltage magnitudes basically corresponding to healthy and faulty conditions respectively. In the bottom part of the table P_{max1} gives the maximum power that can be transferred through the line without compensation.

Table 1: Spreadsheet solution

rl	xl	z	ang z	b_c
0.2731	0.5681	0.6303	1.1227	0.309625

yline	Ang yline	gline	bline
1.5866	-1.1227	0.6875	-1.4299

G_{11}	B_{11}	G_{12}	B_{12}	V_2	Genang
0.6875	-1.1203	-0.6875	1.4299	1	0

P_1	power factor	Q_1	Q_{comp}	Q_{total}	$\frac{-b+\sqrt{b^2-4ac}}{2a}$	$V_{lagupper}$	Ang $V_{lagupper}$	$V_{laglower}$	Ang $V_{laglower}$
0	0.9	0.0000	-0.3096	-0.3096	1.0000	1.0000	0.0000	0.2356	0.3459
-0.1	0.9	-0.0484	-0.2087	-0.2571	1.0000	1.0000	-0.0712	0.2099	0.0461
-0.2	0.9	-0.0969	-0.0980	-0.1949	1.0000	1.0000	-0.1455	0.2125	-0.3070
-0.3	0.9	-0.1453	0.0237	-0.1216	1.0000	1.0000	-0.2236	0.2463	-0.6163
-0.4	0.9	-0.1937	0.1587	-0.0350	1.0000	1.0000	-0.3070	0.3055	-0.8306
-0.45	0.9	-0.2179	0.2321	0.0142	1.0000	1.0000	-0.3512	0.3425	-0.9052
-0.4678	0.9	-0.2266	0.2594	0.0328	1.0000	1.0000	-0.3675	0.3568	-0.9274
-0.5	0.9	-0.2422	0.3102	0.0681	1.0000	1.0000	-0.3976	0.3839	-0.9626
-0.6000	0.9	-0.2906	0.4837	0.1931	1.0000	1.0000	-0.4985	0.4795	-1.0377
-0.6287	0.9	-0.3045	0.5388	0.2343	1.0000	1.0000	-0.5302	0.5105	-1.0515
-0.7	0.9	-0.3390	0.6898	0.3507	1.0000	1.0000	-0.6163	0.5957	-1.0731
-0.75	0.9	-0.3632	0.8120	0.4488	1.0000	1.0000	-0.6856	0.6649	-1.0785
-0.8	0.9	-0.3875	0.9557	0.5683	1.0000	1.0000	-0.7673	0.7466	-1.0752
-0.8500	0.9	-0.4117	1.1402	0.7285	1.0000	1.0000	-0.8732	0.8517	-1.0603
-0.9000	0.9	-0.4359	1.4985	1.0626	1.0595	1.0293	-1.0465	1.0293	-1.0465

P_{max} calculation

a'	b'	c'
-2.4796	12.3850	6.3364

$b'^2-4a'c'$	$\sqrt{b'^2-4a'c'}$	$-b'+\sqrt{b'^2-4a'c'}$	P_{max1}	$-b'-\sqrt{b'^2-4a'c'}$	P_{max2}
216.2333	14.7049	2.3199	-0.4678	-27.0898	5.4626

ABCD Constants Solution

Alternatively the compensation can be obtained using the ABCD constants approach.

The voltage at the sending end bus bar V_2 is given by: -

$$V_2\angle\theta_2 = AV_1\angle(\alpha+\theta_1)+BI_1\angle(\beta+\phi_1)$$

Where A and B and the magnitudes of the A and B constants respectively

θ_2 is the angle of the voltage at bus bar 2.

θ_1 is the angle of the voltage at bus bar 1.

β is the angle of the B constant and

α is the angle of the A constant.

I_1 is the load current

ϕ_1 is the angle of the current at the load end

The current at the load bus bar

$$I_1\angle\varphi_1 = \frac{V_2\angle\theta_2 - AV_1\angle(\alpha+\theta_1)}{B\angle\beta}$$

The complex power at the load bus bar S1 is given by: -

$$S_1 = P_1 + jQ_1 = (V_1\angle\theta_1)(I_1\angle\varphi_1)^*$$
$$= (V_1\angle\theta_1)\left(\frac{V_2\angle\theta_2 - AV_1\angle(a+\theta_1)}{B\angle\beta}\right)^*$$
$$= \frac{V_2V_1\angle(\beta-\delta)}{B} - \frac{AV_1^2\angle(\beta-a)}{B}$$

Where $\delta = \theta_2 - \theta_1$ is the difference between the voltage angle of the sending end bus bar and the load bus bar.

Resolving into the real and imaginary parts: -

$$P_1 = \frac{V_2V_1}{B}\cos(\beta-\delta) - \frac{AV_1^2}{B}\cos(\beta-a)$$
$$Q_1 = \frac{V_2V_1}{B}\sin(\beta-\delta) - \frac{AV_1^2}{B}\sin(\beta-a) \quad . . . (7)$$

When the voltage regulation of the line is given the magnitude of the voltage that the system should operate at is found as follows: -

$$V_{reg} = \frac{|V_{noload}| - |V_{fullload}|}{|V_{noload}|} \times 100\%$$

From which

$$|V_{noload}| = |V_{fullload}|\left(1 + \frac{V_{reg}}{100}\right) = V_1\left(1 + \frac{V_{reg}}{100}\right)$$

But $|V_{noload}| = \left|\frac{V_2\angle\delta}{A\angle a}\right| = \frac{V_2}{A}$

$$\therefore V_2 = A|V_{noload}| = AV_1\left(1 + \frac{V_{reg}}{100}\right) \quad . . . (8)$$

In particular for zero voltage regulation Vreg is 0% and: -

$$V_2 = AV_1$$

The value of V_2 in equation (8) is used in the active power equation in (7) to solve for the voltage angle. After rearranging the equation for P_1: -

$$\delta = \beta - \cos^{-1}\left[\frac{B}{V_2V_1}\left\{P_1 + \frac{AV_1^2}{B}\cos(\beta-\alpha)\right\}\right]$$

The voltage angle is substituted in the equation for reactive power in (7) to obtain the total reactive power at the load: -

$$Q_{RT} = \frac{V_2V_1}{B}\sin(\beta-\delta) - \frac{AV_1^2}{B}\sin(\beta-a)$$
$$= Q_{load} + Q_{comp}$$

where Q_{load} is the reactive power of the load.

The compensation required is: -

$$Q_{comp} = Q_{RT} - Q_{load} . \quad . . . (9)$$

A computer program to solve for the compensation was written in the Matlab language. For a given power factor the power was increased from no load to the maximum value that could be transferred though the transmission line. At each loading condition the compensation to satisfy the specified sending end and receiving end voltage magnitudes was determined.

Special cases to consider are the compensation required at no load to hold the voltage down and that at full load to boost the voltage. These two values determine the range and therefore the sizing of the compensation.

3. Results of the Power System Performance without Compensation

The results of the voltage profile of the medium length line of 150 km are given in Figure 2. The sending end voltage was taken to be 1 per unit in all cases. For each value of power, up to the maximum value that the line can deliver, two voltages were determined, solutions of equation (3), corresponding to healthy and faulty conditions. For example the power delivered by the transmission network is zero on open circuit and when short circuited.

At no load the voltage at the load end is equal to sending end voltage. As the load increases the voltage magnitudes reduces with the rate of reduction dependent on the power factor. There is however a maximum limit of power that the line can deliver at a given power factor. After that limit the voltage infact collapses [1].

It is seen that the voltage profile is in general good for the practical loading of the line of up to 1.8 per unit, especially for leading and unity power factors. The transmission system could be operated up to its thermal limit of 90 MVA.

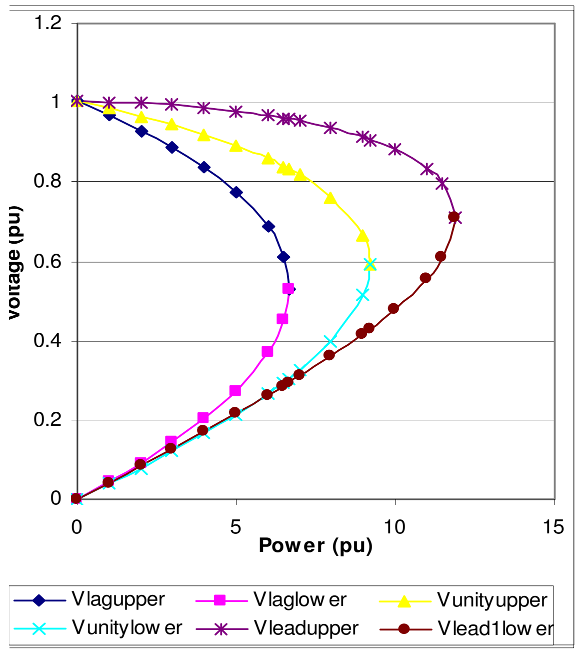


Figure 2: Variation of voltage with power at the end of a medium line

With reference to the voltage profile for the unity power factor, the effect of the lagging power factor is to depress the voltages while the effect of the leading power factor is to improve the voltage regulation. The theoretical maximum power that the transmission network can deliver is far in excess of the rating of the transmission line conductors at any of the three power factors.

Figure 3 shows the voltage profile for the length of 498 km. The line is represented by its nominal π equivalent circuit as the inaccuracies do not affect the overall behaviour of the transmission network. There are two main characteristics; the voltage at the receiving end at no load is much higher than the voltage at the sending end, and the maximum power that the line can deliver is much lower than for the medium line.

The open circuit voltage at the receiving end is 20% higher than the sending end voltage, due to the charging voltamperes consumed by the capacitance of the line. However as the load is increased the voltage reduces rapidly due to the series voltage drops across the resistance and reactance of the line

The maximum power that can be delivered by the line without compensation is about a third of the thermal rating of the line at unity power factor. The effect of the power factor is to reduce the maximum power that can be delivered at lagging power factor and increase it at leading power factor. This is because a lagging power factor load increases the voltage drop along the line while a leading power factor load reduces the voltage drop in the line.

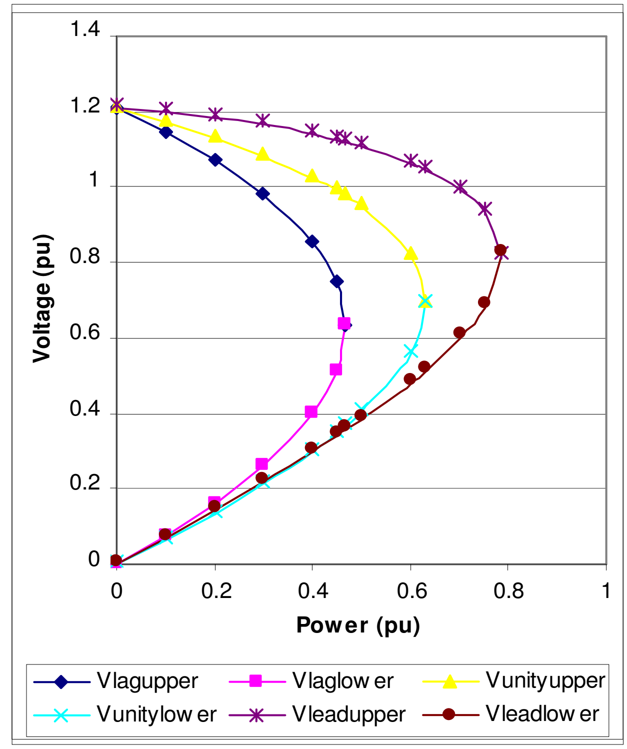


Figure 3: Variation of Voltage with Power – Long line without compensation.

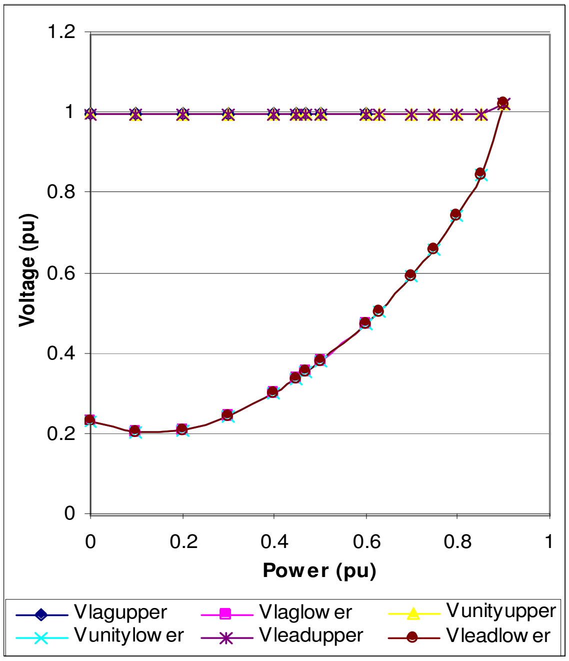
The performance of the system with the long transmission line is unsatisfactory given the large voltage rise at no load and the large variations of voltage with load, as can be seen in Figure 3. In practice compensation would be applied to improve performance.

4. Results of the Power System Performance with Shunt Compensation – Long Line

Figure 4 shows the performance of the long transmission line with compensation. The receiving end voltage is kept at 1 per unit by adjusting the compensation at the receiving end. The maximum power that can be transferred through the line increases to 0.9 per unit, 50% of the thermal rating of the line.

The value of the compensation that is applied to get the voltage profile in figure 4 is given in Figure 5.

The compensation varies from reactance at low loads to capacitive at high loads. The effect of power factor is that at lagging power factor the load requires a lot more capacitive compensation than a load at a leading power factor. For both cases of lagging and leading power factors the total reactive power delivered by the line, the sum of the reactive power of the load and the compensation, is equal to the compensation that is taken by the line at unity power factor for the particular value of active power.



Figue 4: Variation of Voltage with Power-Long line with Compensation

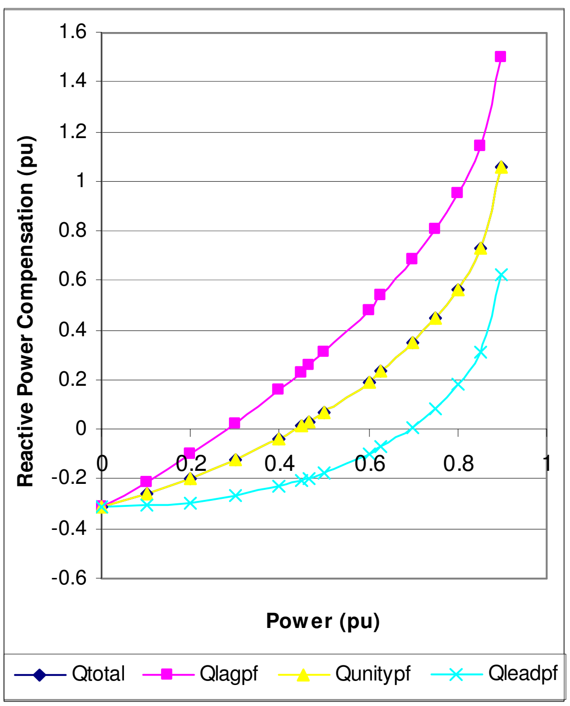


Figure 5: Variation of Compensation with Power for long line

5. Conclusion

The two methods that have been used to determine the shunt compensation are simple and easy to apply. Once the spreadsheet solution is set up the required compensation is determined by successively varying the value of compensation until the specified voltage magnitude at the receiving end bus bar is reached. In this approach both the 'healthy' and 'faulty' voltage magnitudes are determined. In the other approach, the 'ABCD constant method', one merely specifies the required sending end and receiving end voltage magnitudes and the required compensation is calculated.

A two bus bar system had been used to demonstrate the effect of the length of line on the maximum power that can be transferred through the line. While a medium length line has a good voltage profile, even without compensation, that is not the case for a long line. At no load a long line has an appreciable voltage rise at the receiving end. The voltage regulation for a long line is large and usually compensation is required to limit the voltage rise when the load is switched off.

In the particular practical transmission line that was considered the power transfer through the line can be improved to near its thermal limit by application of shunt compensation.

The results are in agreement with the general theory that application of compensation on a long transmission line increases the power that can be transferred through the line; up to the maximum value that can be transferred.

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